PARACONTROLLED CALCULUS AND SOME EXTENSIONS

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We will present the main idea of the paracontrolled calculus, which was recently introduced in the Euclidean situation by Gubinelli, Imkeller and Perkowski. This gives an alternative approach to Hairer's theory in order to deal with singular PDEs. The prototype of the PDEs that we consider is the Parabolic Anderson Model (PAM) on $\mathbb{R}^n$, so to find a solution $u\colon \mathbb{R}^n \times [0,T] \to \mathbb{R}$ of

$$\partial_t u + (-\Delta)u = u \cdot \xi$$

where ξ is the (time-independent) white noise on the Euclidean space. The main difficulty is to give a suitable sense to the product $u \cdot \xi$ since the white noise is too much singular in order to give a sense with (only) analytic tools. After having explained the idea of the paracontrolled calculus, we will then explain how we can extend it in many various situations, where $-\Delta$ is replace by any elliptic operator generating a semigroup with gradient estimates and describe how one can use it for solving the $2-3D$ Parabolic Anderson Model. Also, we will try to give an idea of the higher order paracontrolled calculus, which allows us to 'solve' the main difficulty up to understand the probabilistic step of 'renormalization'. This is a joint work with Ismael Bailleul and Dorothee Frey.