YAGLOM LIMIT FOR STABLE PROCESSES IN CONES

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Let $0 < \alpha < 2$, $d = 1, 2, \ldots$, and let $X = \{X_t, t \geq 0\}$ be the isotropic α-stable Lévy process in $\mathbb{R}^d$. We denote by $\mathbb{P}_x$ the law of the process starting from $x \in \mathbb{R}^d$. Let $\emptyset \neq \Gamma \subset \mathbb{R}^d$ be an arbitrary open Lipschitz cone with vertex at the origin 0. We define

$$\tau_\Gamma = \inf\{t > 0 : X_t \notin \Gamma\}\,, \tag{1}$$

the time of the first exit of X from Γ. The following measure μ will be called the Yaglom limit for X and Γ.

Theorem 1. *There is a probability measure μ concentrated on Γ such that for every Borel set $A \subset \mathbb{R}^d$,*

$$\lim_{t\to\infty} \mathbb{P}_x\left(\frac{X_t}{t^{1/\alpha}} \in A \big| \tau_\Gamma > t\right) = \mu(A)\,, \quad x \in \Gamma\,. \tag{2}$$

The result is proved in [2]. The above condition $\tau_\Gamma > t$ means that X stays, or survives, in Γ for time longer than t. Theorem 1 asserts that, given its survival, X_t rescaled by $t^{1/\alpha}$ has a limiting distribution independent of the starting point. We note that rescaling is essential for the limit to be nontrivial. The Yaglom limit μ corresponds with the idea of "quasi-stationarity", as expressed by Bartlett [1]:

> It still may happen that the time to extinction is so long that it is still of more relevance to consider the effectively ultimate distribution (called a quasi-stationary distribution) [...]

We also construct and estimate entrance laws for the process from the vertex into the cone. Our approach relies on the scalings of the stable process and the cone, which allow to express the temporal asymptotics of the distribution of the process at infinity by means of the spatial asymptotics of harmonic functions of the process at the vertex; on the representation of the probability of survival of the process in the cone as a Green potential; and on the approximate factorization of the heat kernel of the cone, which secures compactness and yields a limiting (Yaglom) measure by means of Prokhorov's theorem.

References

[1] M. S. Bartlett. *Stochastic population models in ecology and epidemiology.* Methuen's Monographs on Applied Probability and Statistics. Methuen& Co., Ltd., London; John Wiley& Sons, Inc., New York, 1960.

[2] Krzysztof Bogdan, Zbigniew Palmowski, and Longmin Wang, *Yaglom limit for stable processes in cones*, Electron. J. Probab. **23** (2018), 19 pp.