NESTED OCCUPANCY SCHEMES IN RANDOM ENVIRONMENTS

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Let $(P_r)_{r\in\mathbb{N}}$ be a collection of positive random variables satisfying $\sum_{r\geq 1} P_r = 1$ a.s. Assume that, given $(P_r)_{r\in\mathbb{N}}$, 'balls' are allocated independently over an infinite collection of 'boxes' 1, 2, ... with probability P_r of hitting box r, $r\in\mathbb{N}$. The occupancy scheme arising in this way is called the *infinite occupancy scheme in the random environment* $(P_r)_{r\geq 1}$.

A popular model of the infinite occupancy scheme in the random environment assumes that the probabilities $(P_r)_{r\in\mathbb{N}}$ are formed by an enumeration of the a.s. positive points of

$$\{e^{-X(t-)}(1-e^{-\Delta X(t)}) : t\geq 0\}, \tag{1}$$

where $X := (X(t))_{t\geq 0}$ is a subordinator (a nondecreasing Lévy process) with $X(0) = 0$, zero drift, no killing and a nonzero Lévy measure, and $\Delta X(t)$ is a jump of X at time t. Since the closed range of the process X is a regenerative subset of $[0,\infty)$ of zero Lebesgue measure, one has $\sum_{r\geq 1} P_r = 1$ a.s. When X is a compound Poisson process, collection (1) transforms into a *residual allocation model*

$$P_r := W_1 W_2 \cdot \ldots \cdot W_{r-1}(1-W_r), \quad r\in\mathbb{N}, \tag{2}$$

where W_1, $W_2, \ldots$ are i.i.d. random variables taking values in $(0,1)$.

Next, I define a nested infinite sequence of the infinite occupancy schemes in random environments. This means that I construct a nested sequence of environments (random probabilities) and the corresponding 'boxes' so that the same collection of 'balls' is thrown into all 'boxes'. To this end, I use a weighted branching process with positive weights which is nothing else but a multiplicative counterpart of a branching random walk.

The nested sequence of environments is formed by the weights $(R(u))_{|u|=1} = (P_r)_{r\in\mathbb{N}}$, $(R(u))_{|u|=2}, \ldots$, say, of the subsequent generations individuals in a weighted branching process. Further, I identify individuals with 'boxes'. At time $j = 0$, all 'balls' are collected in the box $\oslash$ which corresponds to the initial ancestor. At time $j = 1$, given $(R(u))_{|u|=1}$, 'balls' are allocated independently with probability $R(u)$ of hitting box u, $|u| = 1$. At time $j = k$, given $(R(u))_{|u|=1}, \ldots, (R(u))_{|u|=k}$, a ball located in the box u with $|u| = k$ is placed independently of the others into the box ur, $r\in\mathbb{N}$ with probability $R(ur)/R(u)$.

Assume that there are n balls. For $r = 1, 2, \ldots, n$ and $j\in\mathbb{N}$, denote by $K_{n,j,r}$ the number of boxes in the jth generation which contain exactly r balls and set

$$K_{n,j}(s) := \sum_{r=\lceil n^{1-s}\rceil}^{n} K_{n,j,r}, \quad s\in[0,1],$$

where $x\mapsto \lceil x\rceil = \min\{n\in\mathbb{Z} : n\geq x\}$ is the ceiling function. With probability one the random function $s\mapsto K_{n,j}(s)$ is right-continuous on $[0,1)$ and has finite limits from the left on $(0,1]$ and as such belongs to the Skorokhod space $D[0,1]$. I am going to present sufficient conditions which ensure functional weak convergence of $(K_{n,j_1}(s), \ldots, K_{n,j_m}(s))$, properly normalized and centered, for any finite collection of indices $1\leq j_1 < \ldots < j_m$ as the number n of balls tends to ∞. If time permits, I shall discuss specializations of the general result to $(P_r)_{r\in\mathbb{N}}$ given by (1) and (2).

The talk is based on a work in progress, joint with Sasha Gnedin (London).