MAX-LINEAR MODELS ON INFINITE GRAPHS GENERATED BY BERNOULLI BOND PERCOLATION

CLAUDIA KLÜPPELBERG

We extend finite-dimensional max-linear models to models on infinite graphs, and investigate their relations to classical percolation theory, more precisely to nearest neighbor bond percolation. We focus on the plane square lattice $\mathbb{Z}^2$ with edges to the nearest neighbours, where we direct all edges in a natural way (to the right or up) resulting in a directed acyclic graph (DAG) on $\mathbb{Z}^2$. On this infinite DAG a random sub-DAG may be constructed by choosing vertices and edges between them at random. In a Bernoulli bond percolation DAG edges are independently declared open with probability $p \in (0,1)$ and closed otherwise. The random DAG consists then of the vertices and the open directed edges.

We find for the subcritical case where $p \leq 1/2$ that two random variables of the max-linear model become independent with probability 1, whenever their distance tends to infinity. In contrast, for the supercritical case where $p > 1/2$ two random variables are dependent with positive probability, even when their node distance tends to infinity.

We also consider changes in the dependence properties of random variables on a sub-DAG H of a finite or infinite graph on $\mathbb{Z}^2$, when enlarging this subgraph. The method of enlargement consists of adding nodes and edges of Bernoulli percolation clusters. Here we start with X_i and X_j independent in H, and answer the question, whether they can become dependent in the enlarged graph. We evaluate critical probabilities such that X_i and X_j become dependent in the enlarged graph with positive probability or with probability 1. We find in particular that for every DAG H with finite number of nodes, in the enlarged graph, X_i and X_j remain independent with positive probability. On the other hand, if H has nodes $\mathbb{Z}^2$ and percolates everywhere; i.e. every connected component of H is infinite, then X_i and X_j become dependent with probability 1 in the enlarged graph.

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