NORMS OF INHOMOGENOUS RANDOM MATRICES

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The spectral norm of any symmetric matrix is bigger than the largest Euclidean norm of its rows. We show that for Gaussian matrices with independent entries this obvious bound may be reversed in average up to a universal constant.

Theorem 1. Let $X = (X_{ij})_{i,j \leq d}$ be a symmetric matrix such that $(X_{ij})_{i \leq j}$ are independent centered Gaussian random variables. Then

$$\mathbb{E}||X|| \le C \mathbb{E} \max_{i} \sqrt{\sum_{j} X_{ij}^2}.$$

We also discuss how to estimate quantities in the above theorem by an explicit expression in terms of the variances of the matrix entries and present similar result for Schatten norms.

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