Testing independence of random elements with the distance covariance

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This is joint work with Herold Dehling (Bochum), Muneya Matsui (Nagoya), Gennady Samorodnitsky (Cornell) and Laleh Tafakori (Melbourne). Distance covariance was introduced by Székely, Rizzo and Bakirov (2007) as a measure of dependence between vectors of possibly distinct dimensions. Since then it has attracted attention in various fields of statistics and applied probability. The distance covariance of two random vectors X, Y is a weighted L^2 distance between the joint characteristic function of (X, Y) and the product of the characteristic functions of X and Y. It has the desirable property that it is zero if and only if X, Y are independent. This is in contrast to classical measures of dependence such as the correlation between two random variables: zero correlation corresponds to the absence of linear dependence but does not give any information about other kinds of dependencies. We consider the distance covariance for stochastic processes X, Y defined on some interval and having square integrable paths, including Lévy processes, fractional Brownian, diffusions, stable processes, and many more. Since distance covariance is defined for vectors we consider discrete approximations to X, Y. We show that sample versions of the discretized distance covariance converge to zero if and only if X, Y are independent. The sample distance covariance is a degenerate V-statistic and, therefore, has rate of convergence which is much faster than the classical $\sqrt{n}$-rates. This fact also shows nicely in simulation studies for independent X, Y in contrast to dependent X, Y.