DEGENERATE POINCARE-SOBOLEV INEQUALITIES: RECENT RESULTS

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Recent results on degenerate Poincaré-Sobolev inequalities.

In this lecture I plan to discuss some recent results obtained with E. Rela concerning Poincaré and Poincaré-Sobolev inequalities with weights. These results improve some classical estimates due to Fabes-Kenig-Serapioni obtained in the 80's in connection with the local regularity of weak solutions of degenerate elliptic equations. I will also show that there is a connection with the Keith-Zhong phenomenon using extrapolation ideas.

Theorem 1. *Given $1 \leq p < n$ and $w \in A_p$ we define p^* as the "degenerate" Poincaré-Sobolev exponent defined by*

$$\frac{1}{p} - \frac{1}{p^*} = \frac{1}{n(p + \log[w]_{A_p})}. \tag{1}$$

Then the following Poincarée-Sobolev inequality holds,

$$\left(\frac{1}{w(Q)}\int_Q |f - f_Q|^{p^*}\, w\, dx\right)^{\frac{1}{p^*}} \leq c_n [w]_{A_p}^{\frac{1}{p}} \ell(Q) \left(\frac{1}{w(Q)}\int_Q |\nabla f|^p\, w\, dx\right)^{1/p}$$