3D POSITIVE LATTICE WALKS AND SPHERICAL TRIANGLES

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In this talk we explore the asymptotic enumeration of three-dimensional excursions confined to the positive octant. We focus on the critical exponent, which admits a universal formula in terms of the principal Dirichlet eigenvalue of a certain spherical triangle, itself being characterized by the steps of the model. Our main objective is to relate combinatorial properties of the step set (structure of the so-called group of the walk, existence of a Hadamard factorization, existence of differential equations satisfied by the generating functions) to geometric or analytic properties of the associated spherical triangle (remarkable angles, tiling properties, existence of an exceptional closed-form formula for the principal eigenvalue). As in general the eigenvalues of the Dirichlet problem on a spherical triangle are not known in closed form, we also develop a finite-elements method to compute approximate values, typically with a 10^{-10} precision.