RIESZ DISTRIBUTIONS AND BETA DISTRIBUTIONS IN THE RATIONAL DUNKL SETTING

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Riesz distributions play an important role in the analysis on symmetric cones, tracing back to fundamental work by Gindikin in the 1970ies. In this talk, we study Riesz distributions and beta distributions in the Dunkl setting of type A, as well as an associated Laplace transform. These objects are closely related to a one-parameter generalization of the theory of hypergeometric functions of matrix argument which was introduced by Macdonald in the 1980ies, partly on a just formal level, and posted on the arviv [1] only some years ago.

We give the Dunkl-type Laplace transform a rigorous foundation and then focus on the question for which parameters Riesz and beta distributions in the Dunkl setting are actually measures. For Riesz distributions on symmetric cones, the admissible range of parameters is given by the so-called Wallach set. Analogous questions for beta distributions were recently studied in [2] in relation with Sonine formulas for Bessel functions on symmetric cones. In the Dunkl case, a natural generalization of the Wallach set enters the picture, and the question when beta distributions are actually measures is closely related to the existence of Sonine-type integral representations between Dunkl kernels with different multiplicities. As a consequence, we obtain examples where the intertwining operator between Dunkl operators associated with multiplicities $k \geq 0$ and $k' \geq k$ is not positive, which disproves a long-standing conjecture.

Part of this talk is based on joint work with Michael Voit.

References:

[1] I.G. Macdonald, Hypergeometric Functions I. ArXiv 1309.4568.

[2] M. Rösler, M. Voit, Beta distributions and Sonine integrals for Bessel functions on symmetric cones. ArXiv 1801.07304. To appear in *Stud. Appl. Math.*