REMARKS ON LOCALIZED SHARP FUNCTIONS ON CERTAIN SETS IN $\mathbb{R}^n$

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On $\mathbb{R}^n$ let $f^{\#}_{\Delta}(x)$ and $M^{\Delta}f(x)$ denote the classical dyadic sharp function and dyadic maximal function respectively, that is,

$$f^{\#}_{\Delta}(x) = \sup_{x\in Q\in\Delta} \frac{1}{|Q|}\int_Q |f(y) - f_Q|\,dy, \qquad M^{\Delta}f(x) = \sup_{x\in Q\in\Delta} \frac{1}{|Q|}\int_Q |f(y)|\,dy,$$

where Δ denotes the collection of all dyadic cubes in $\mathbb{R}^n$ and f_Q is the average of f over Q. Suppose that $f \in L^{p_0}(\mathbb{R}^n)$ for some p_0. The well-known Fefferman-Stein inequality asserts that if $1 < p < \infty$, $1 \le p_0 \le p$, and $f^{\#}_{\Delta} \in L^p(\mathbb{R}^n)$, then $M^{\Delta}f \in L^p(\mathbb{R}^n)$ and

$$\|M^{\Delta}f\|_{L^p(\mathbb{R}^n)} \le C_n(p)\|f^{\#}_{\Delta}\|_{L^p(\mathbb{R}^n)} \tag{1}$$

Let Ω be a domain in $\mathbb{R}^n$. Our goal is to define for $f \in L^1_{\rm loc}(\Omega)$ a localized version $f^{\#}_{\rm loc}$ of the sharp function which will satisfy the analogue of the Fefferman–Stein inequality. By localized we mean that the cubes which are taken in the definition of $f^{\#}_{\rm loc}(x)$ are contained in a bounded set $\mathcal{B}_x \subset \Omega$. So one possible definition can be taken as follows. Let $\tau : \Omega \to (0,\infty)$. For $f \in L^1_{\rm loc}(\Omega)$ we set

$$f^{\#}_{{\rm loc},\,\tau}(x) = \sup_{x\in Q\subset\Omega,\ \ell(Q)<\tau(x)} \frac{1}{|Q|}\int_Q |f(y) - f_Q|\,dy,$$

where Q is any cube (not necessarily dyadic) and $\ell(Q)$ denotes its side-length. Our aim is to show that for certain sets Ω in $\mathbb{R}^n$ if $\tau(x)$ behaves like $\frac{1}{2^n}{\rm dist}(x,\partial\Omega)$, then $f^{\#}_{{\rm loc},\,\tau}$ satisfies (1).These were obtained by proving modifications of the good lambda inequality.

This talk is based on joint work with Jacek Dziubański.