THE DECAY RATE OF FARMONIC FUNCTIONS FOR NON-SYMMETRIC STRICTLY α-STABLE LÉVY PROCESSES

TOMASZ JUSZCZYSZYN

The boundary Harnack inequality is a statement about positive harmonic functions in an open set D, which are equal to zero on a part of the boundary. It states that if D is regular enough, z is a boundary point of D, f and g are positive and harmonic in D, and both f and g converge to 0 on $\partial D \cap B(z, R)$, then for every $r \in (0, R)$

$$\sup_{x \in D \cap B(z,r)} \frac{f(x)}{g(x)} \le c_{BHI} \inf_{x \in D \cap B(z,r)} \frac{f(x)}{g(x)}, \tag{1}$$

where constant c_{BHI} depends only on D and r.

Over last 20 years BHI was shown to hold for various types of stochastic processes and domains. What often followed was the existence of the limits of the ratios of harmonic functions as $r \to 0$ in (**??**) and their explicit decay rate.

Results that are presented in my talk are about the decay rate of f next to the boundary of D for non-symmetric strictly α-stable Lévy processes. We assume that the Lévy measure of X has a density function which is Hölder continuous on the unit sphere with exponent ϵ and for domains D of $C^{1,1}$ class for $\alpha < 1$ and of $C^{2,\alpha+\epsilon-1}$ class for $1 \le \alpha < 2$.

Theorem 1. *Let $z \in \partial D \cap B(z, R_0)$ and let f be a non-negative function which is harmonic in $D \cap B(z, R_0)$ and vanishes continuously on $D^c \cap B(z, R_0)$, then*

$$\lim \frac{f(x)}{\delta_D(x)^{\beta(x)}} \text{ exists as } x \to z, x \in D,$$

where $\beta(x) = \alpha \mathbb{P}^0(\langle X_t, n(x) \rangle > 0)$, vector $n(x)$ is a normal vector to the boundary of D at the point $z(x)$ — orthogonal projection of x onto the boundary of D.