EXTENSION THEOREM FOR NONLOCAL OPERATORS

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We present an extension theorem related to spaces generated by quadratic forms

$$\mathcal{E}_D(u,u) = \iint_{(D^c \times D^c)^c} (u(x) - u(y))^2 \nu(x,y)\, dx\, dy.$$

Here D is an open subset of $\mathbb{R}^d$, and ν is a Lévy-type kernel. To establish our results, we use objects and methods from the potential theory, in particular the communication kernel

$$\gamma_D(z,w) = \int_D P_D(x,w)\nu(z,x)\, dx, \quad z, w \in D^c,$$

where P_D is the Poisson kernel of D.

Our extension theorem states that if squared increments of a function g given on D^c are integrable with weight γ_D, then the harmonic function, given for $x \in D$ by the Poisson integral $\int_{D^c} g(z) P_D(x,z)\, dz$, serves as the extension of g, i.e. its $\mathcal{E}_D$ form is finite.

We also provide estimates for the kernel γ_D and applications to the Dirichlet problem for nonlocal operators associated with ν.

The talk is based on joint work [1] with Krzysztof Bogdan, Tomasz Grzywny and Katarzyna Pietruska-Pałuba.

References

[1] Bogdan K., Grzywny T., Pietruska-Pałuba K., Rutkowski A., *Extension theorem for nonlocal operators*, arXiv:1710.05580.