

GREEN FUNCTIONS ON REAL HYPERBOLIC SPACE

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We consider the real hyperbolic space $\mathbb{H}^n = \{x \in \mathbb{R}^n : x_n > 0\}$, where the Laplace-Beltrami operator is given by

$$\Delta_{\mathbb{H}^n} = \frac{1}{2}x_n^2 \sum_{k=1}^n \frac{\partial^2}{\partial x_k^2} - \frac{n-2}{2}x_n \frac{\partial}{\partial x_n}.$$

A survey on results concerning Green functions and other aspects of potential theory on the space will be presented. This includes some general representations and relationships as well as both-sided estimates in case of special sets. In particular, we will focus on uniform estimates on Green function of any ball that have been recently derived by M. Ryznar, G. Serafin and T. Žak. Although balls are one of the most important sets in the theory, only results for those with a fixed radius have been known so far.

Most of the result have been obtained due to the probabilistic approach exploiting hyperbolic Brownian motion, which is a process generated by $\Delta_{\mathbb{H}^n}$. Its relationships to Bessel processes and Brownian motion killed by a suitable potential have played a crucial role at many points.