TAIL INDICES FOR AN AFFINE STOCHASTIC RECURSION WITH TRIANGULAR MATRICES

WITOLD ŚWIĄTKOWSKI

We study the solution W to the stochastic equation $W \stackrel{d}{=} AW + B$, where the distribution of $(A, B) \in M_{d\times d}(\mathbb{R}_+) \times \mathbb{R}_+^d$ is given and it satisfies some integrability assumptions.

It was proved by Kesten that if the matrix A^n has all entries positive for some n, then each coordinate W_i of the solution has a regularly varying tail, with a common tail index. On the other hand, if A is a diagonal matrix, the tail indices clearly can be different for each coordinate. It is natural to ask, what happens in non-trivial cases that do not fit into Kesten's setting.

We assume that A is a triangular matrix. It does not satisfy the Kesten's assumptions since A^n is also triangular. We develop methods to prove that

$$\lim_{t\to\infty} t^{\tilde{\alpha}_i}\mathbb{P}(W_i > t) = c_i.$$

The tail indices $\tilde{\alpha}_i$ depend on the laws of the diagonal entries A_{jj} with $j \geq i$ and on positions of zero entries of the matrix A. They are given by an exact expression. The constants c_i are also calculated.

The talk is based on a joint work with Muneya Matsui.