HEAT KERNELS OF NON-SYMMETRIC LÉVY-TYPE OPERATORS

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We construct the fundamental solution (the heat kernel) p^κ to the equation $\partial_t = \mathcal{L}^\kappa$, where under certain assumptions the operator $\mathcal{L}^\kappa$ takes one of the following forms,

$$\mathcal{L}^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<1} \langle z, \nabla f(x) \rangle) \kappa(x,z) J(z) \, dz \,,$$

$$\mathcal{L}^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x)) \kappa(x,z) J(z) \, dz \,,$$

$$\mathcal{L}^\kappa f(x) := \frac{1}{2} \int_{\mathbb{R}^d} (f(x+z) + f(x-z) - 2f(x)) \kappa(x,z) J(z) \, dz \,.$$

In particular, $J\colon \mathbb{R}^d \to [0,\infty]$ is a Lévy density, i.e., $\int_{\mathbb{R}^d} (1 \wedge |x|^2) J(x) dx < \infty$. The function $\kappa(x,z)$ is assumed to be Borel measurable on $\mathbb{R}^d \times \mathbb{R}^d$ satisfying $0 < \kappa_0 \leq \kappa(x,z) \leq \kappa_1$, and $|\kappa(x,z) - \kappa(y,z)| \leq \kappa_2 |x-y|^\beta$ for some $\beta \in (0,1)$. We prove the uniqueness, estimates, regularity and other qualitative properties of p^κ. It gives rise to a Feller semigroup $(P_t)_{t \geq 0}$ and a Feller process $X = (X_t, \mathbb{P}^x)$ that in turn solves the martingale problem for $(\mathcal{L}^\kappa, C_c^\infty(\mathbb{R}^d))$.

The talk is based on a joint work with T. Grzywny.