

NON-INTEGRABILITY OF THE SASANO SYSTEM OF TYPE $D_5^{(1)}$

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In [1] Yusuke Sasano introduce higher order Painlevé system with affine Weyl group symmetry of type $D_l^{(1)}$. In this talk we study the integrability of the Sasano system which is invariant under the extended affine Weyl group $\widetilde{W}(D_5^{(1)})$. This system is expressed as the Hamiltonian system

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}, \quad \frac{dy}{dt} = -\frac{\partial H}{\partial x}, \quad \frac{dz}{dt} = \frac{\partial H}{\partial w}, \quad \frac{dw}{dt} = -\frac{\partial H}{\partial z}$$

with the Hamiltonian

$$H = H_V(x, y, t; \alpha_2 + \alpha_5, \alpha_1, \alpha_2 + 2\alpha_3 + \alpha_4) + H_V(z, w, t; \alpha_5, \alpha_3, \alpha_4) + \frac{2yz((z-1)w + \alpha_3)}{t},$$

where by $H_V(q, p, t; \gamma_1, \gamma_2, \gamma_3)$ is denoted the Hamiltonian associated with the Painlevé V equation, i.e.

$$H_V(q, p, t; \gamma_1, \gamma_2, \gamma_3) = \frac{q(q-1)p(p+t) - (\gamma_1 + \gamma_3)qp + \gamma_1 p + \gamma_2 tq}{t}.$$

The complex parameters $\alpha_j, 0 \leq j \leq 5$ satisfy the relation

$$\alpha_0 + \alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 + \alpha_5 = 1.$$

In this talk we will present a rigorous proof that when

$$\alpha_1 = \alpha_2 = \alpha_3 = 0, \quad \alpha_4 = 1, \quad \alpha_0 = -\alpha_5$$

the Sasano system of type $D_5^{(1)}$ is not integrable in the sense of the Hamiltonian dynamics by meromorphic first integrals which are rational functions in t . To obtain this result we utilize the Morales-Ramis-Simó theory of non-integrability of analytic Hamiltonian systems. In addition, with the aid of the Bäcklund transformations of the Sasano system we extend this non-integrable result to the entire orbit of the parameters $\alpha_j, 0 \leq j \leq 5$.

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REFERENCES

- [1] Y. Sasano, *Higher order Painlevé equations of type $D_5^{(1)}$* , RIMS Kokyuroku **1473** (2006), pp. 143-163