following the argument in [2], Section 3, Remark (5). However, a more generally formulated theorem would perhaps be desirable — which gave a necessary and sufficient condition appropriately involving domains.

Finally we would like to mention that although the singular sequence problem may remain of interest for normal and some normal-like operators also, it is not of interest very much beyond them. To show this we would like to give a short construction which might be of some use elsewhere in these considerations. Let B be any bounded noncompact operator such that $\sigma_e(B) = 0$ and $B+I$ is invertible. Let A be B^2. It follows immediately (from $A+B = (B+I)B$) that $\sigma_e(A) = \sigma_e(A+B) = 0$ and that $A+B$ and A have the same singular sequences. But B was not compact. One may use for an explicit example the $B = \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right)$ of [2]. In that case one even has A $(=0)$ self-adjoint, $ss_{A+B}(\lambda) = ss_A(\lambda)$, and a negative conclusion for the singular sequence question.

Added in proof. For W^* algebra the question of Zemánek [11] may be answered affirmatively, and negatively for general C^* algebras. For further references see K. Gustafson and M. Seddighin, *Nonperturbing algebras*, to appear, and the survey by the present author to appear in the proceedings of the Spectral Theory Semester (2nd), 1982.

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ON THE SPECTRAL PROPERTIES OF TENSOR PRODUCTS OF LINEAR OPERATORS IN BANACH SPACES*

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1. Introduction

Let X and Y be complex Banach spaces and let α be a uniform reasonable norm on $X\otimes Y$. The completion of $X\otimes Y$ with respect to α is denoted by $X\hat{\otimes}_\alpha Y$. Let $A\colon D[A]\subset X\to X$ and $B\colon D[B]\subset Y\to Y$ be densely defined closed linear operators with nonempty resolvent sets. Associated with each polynomial of degrees m in ξ and n in η

$$P(\xi,\eta) = \sum_{jk} c_{jk}\xi^j\eta^k \tag{1.1}$$

is a polynomial operator

$$P\{A\otimes I, I\otimes B\} \equiv \sum_{jk} c_{jk}A^j\otimes B^k \tag{1.2}$$

in $X\hat{\otimes}_\alpha Y$ with domain $D[A^m]\otimes D[B^n]$. In particular, to $\xi+\eta$ and $\xi\eta$ correspond respectively $A\otimes I+I\otimes B$ and $A\otimes B$. The identity operators in both X and Y are denoted by the same I. Assume that (1.2) *is closable in* $X\hat{\otimes}_\alpha Y$ *with closure* $\boldsymbol{P}$. This is the case, for instance, if α is faithful on $X\otimes Y$, i.e. if the natural continuous linear mapping $j_\varepsilon^\alpha\colon X\hat{\otimes}_\alpha Y\to X\hat{\otimes}_\varepsilon Y$ is one-to-one.

We are interested in the problem of what spectral contributions $\boldsymbol{P}$ gets from A and B.

The aim of this note is to make a brief survey of our results ([9], [10], [11], [12]) on the exact representations of the spectrum, essential spectra, approximate point spectrum and approximate deficiency spectrum of $\boldsymbol{P}$ by the parts of the spectra of A and B. By the essential spectra are meant those in the sense of F. E. Browder [3], F. Wolf [22], M. Schechter [18], Gustafson–Weidmann [7] and T. Kato [14]. Further we refer to the formulae expressing the nullity, deficiency and index of $\boldsymbol{P}$ in terms of the quantities concerning A and B.

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The results may amplify the method of separation of variables and serve as basic principles in the spectral theory of many-body Schrödinger operators (cf. Balslev–Combes [1], B. Simon [20]).

If, in addition, the crossnorm α is faithful on $X\otimes Y$, all the results are valid for the closure of another associated polynomial operator

$$P[A\otimes I, I\otimes B] \equiv \sum_{jk} c_{jk} A^j \hat{\otimes}_\alpha B^k \tag{1.3}$$

in $X\hat{\otimes}_\alpha Y$, for (1.2) and (1.3) have the same closure. Here the $A^j\hat{\otimes}_\alpha B^k$ are the closures of the $A^j\otimes B^k$ in $X\hat{\otimes}_\alpha Y$.

2. Preliminaries

For the basic notions used here we follow T. Kato [14].

Let T be a densely defined closed linear operator in a Banach space. The null space of T is denoted by $N[T]$. The spectrum and resolvent set of T are denoted by $\sigma(T)$ and $\varrho(T)$, respectively. The nullity, deficiency and index of T are denoted by $\operatorname{nul} T$, $\operatorname{def} T$ and $\operatorname{ind} T$, and the algebraic multiplicity of an eigenvalue λ of T by $t(T;\lambda)$.

The *Browder essential spectrum* of T, $\sigma_{\mathrm{eb}}(T)$, is $\sigma(T)$ excluding all isolated, finite-dimensional eigenvalues of T.

The *Wolf* (resp. *Kato*) *essential spectrum* of T, $\sigma_{\mathrm{ew}}(T)$ (resp. $\sigma_{\mathrm{ek}}(T)$), is the set of all λ in the complex plane C for which $T-\lambda$ is not Fredholm (resp. semi-Fredholm). The *Schechter essential spectrum* of T, $\sigma_{\mathrm{em}}(T)$, is the union of $\sigma_{\mathrm{ew}}(T)$ and the set of all $\lambda\in C$ for which $T-\lambda$ is Fredholm with $\operatorname{ind}(T-\lambda)\neq 0$.

The approximate nullity and approximate deficiency of T are denoted by $\operatorname{nul}' T$ and $\operatorname{def}' T$. We have $\operatorname{def}' T = \operatorname{nul}' T'$, where T' is the adjoint of T. It is known [14] that $\operatorname{nul}' T$ is positive (resp. infinite) if and only if there is a sequence (resp. a non-compact sequence) $\{z_l\}_{l=1}^{\infty}$ in $D[T]$ of unit vectors such that $Tz_l\to 0$ as $l\to\infty$.

Let

$$\sigma_\pi(T) = \{\lambda\in C;\ \operatorname{nul}'(T-\lambda) > 0\},\qquad \sigma_\delta(T) = \{\lambda\in C;\ \operatorname{def}'(T-\lambda) > 0\},$$
$$\sigma_+(T) = \{\lambda\in C;\ \operatorname{nul}'(T-\lambda) = \infty\},\qquad \sigma_-(T) = \{\lambda\in C;\ \operatorname{def}'(T-\lambda) = \infty\}.$$

$\sigma_\pi(T)$ and $\sigma_\delta(T)$ are respectively the *approximate point spectrum* and *approximate deficiency spectrum* of T. $\sigma_+(T)$ and $\sigma_-(T)$ are also called the *essential spectra* by Gustafson–Weidmann. We have

$$\sigma(T) = \sigma_\pi(T)\cup\sigma_\delta(T),\qquad \sigma_{\mathrm{ew}}(T) = \sigma_+(T)\cup\sigma_-(T),$$
$$\sigma_{\mathrm{ek}}(T) = \sigma_+(T)\cap\sigma_-(T).$$

All the six essential spectra coincide if T is a self-adjoint operator in a Hilbert space.

For a Banach space Z, denote by $I(Z)$ (resp. $H(Z)$) the family of all topological linear isomorphisms (resp. homomorphisms) of Z into itself.

A crossnorm α is said to have the *i*- (resp. *h*-) *property* on $X\otimes Y$ if for every $T\in I(X)$ (resp. $T\in H(X)$) and $S\in I(Y)$ (resp. $S\in H(Y)$) there exist constants $\gamma(T\otimes I) > 0$ and $\gamma(I\otimes S) > 0$ such that

$$\|(T\otimes I)u\|_\alpha \geqslant \gamma(T\otimes I)\operatorname{dist}(u, N[T]\otimes Y),\qquad u\in X\otimes Y,$$

and

$$\|(I\otimes S)u\|_\alpha \geqslant \gamma(I\otimes S)\operatorname{dist}(u, X\otimes N[S]),\qquad u\in X\otimes Y.$$

The injective $\otimes$-norm α and, in particular, the smallest reasonable norm ε has the *i*-property. For both X and Y Hilbert spaces, the greatest reasonable norm π, the smallest reasonable norm ε and the prehilbertian norm σ have the *h*- and *i*-properties; more generally, every $\otimes$-norm in the sense of Grothendieck [6] has these properties (see [12]).

3. The main results

For simplicity, to state our results, we restrict ourselves to the case where $P(\xi,\eta) = \xi+\eta$; the corresponding $\boldsymbol{P}$ is denoted by $\boldsymbol{H}$. Therefore $\boldsymbol{H}$ is the closure of $A\otimes I+I\otimes B$ in $X\hat{\otimes}_\alpha Y$.

In the following theorems it is assumed that the *resolvent sets* $\varrho(A)$ *and* $\varrho(B)$ *of* A *and* B *include, respectively, the outsides of the sectors*

$$S(\theta_A) = \{\xi\in C;\ |\arg\xi|\leqslant\theta_A\}\quad \textit{and}\quad S(\theta_B) = \{\eta\in C;\ |\arg\eta|\leqslant\theta_B\}$$

with $0\leqslant\theta_A+\theta_B<\pi$, *and that*

$$\|\xi(\xi-A)^{-1}\|\leqslant M_A(\arg\xi),\qquad \xi\notin S(\theta_A),$$

and

$$\|\eta(\eta-B)^{-1}\|\leqslant M_B(\arg\eta),\qquad \eta\notin S(\theta_B),$$

where $M_T(\theta)$ *is a constant depending only on* T *and* θ.

The following convention is used. For the subsets σ_A of $\sigma(A)$ and σ_B of $\sigma(B)$, the set $\sigma_A+\sigma_B$ is empty if at least one of the sets σ_A and σ_B is empty.

THEOREM 1. *Let α be a uniform reasonable norm on $X\otimes Y$. Then the following relations hold*:

$$\sigma(\boldsymbol{H}) = \sigma(A)+\sigma(B). \tag{3.1}$$

$$\sigma_{\mathrm{eb}}(\boldsymbol{H}) = \big(\sigma_{\mathrm{eb}}(A)+\sigma(B)\big)\cup\big(\sigma(A)+\sigma_{\mathrm{eb}}(B)\big). \tag{3.2}$$

If λ is outside the set (3.2), *then*

$$t(\boldsymbol{H};\lambda) = \sum_{\substack{(\mu,\nu),\ \mu+\nu=\lambda\\ \mu\in\sigma(A)\setminus\sigma_{\mathrm{eb}}(A)\\ \nu\in\sigma(B)\setminus\sigma_{\mathrm{eb}}(B)}} t(A;\mu)\,t(B;\nu). \tag{3.3}$$

$$\sigma_{\mathrm{ew}}(\boldsymbol{H}) = \big(\sigma_{\mathrm{ew}}(A)+\sigma(B)\big)\cup\big(\sigma(A)+\sigma_{\mathrm{ew}}(B)\big). \tag{3.4}$$

If λ is outside the set (3.4), *then*

$$\operatorname{ind}(\boldsymbol{H}-\lambda) = \sum_{\substack{(\mu,\nu),\ \mu+\nu=\lambda \\ \mu\in\sigma(A)\setminus\sigma_{eb}(A) \\ \nu\in\sigma_{eb}(B)\setminus\sigma_{ew}(B)}} \operatorname{ind}(B-\nu)\sum_{p=1}^{\infty}\big(\operatorname{nul}(A-\mu)^p-\operatorname{nul}(A-\mu)^{p-1}\big)+ \tag{3.5}$$
$$+\sum_{\substack{(\mu,\nu),\ \mu+\nu=\lambda \\ \mu\in\sigma_{eb}(A)\setminus\sigma_{ew}(A) \\ \nu\in\sigma(B)\setminus\sigma_{eb}(B)}} \operatorname{ind}(A-\mu)\sum_{p=1}^{\infty}\big(\operatorname{nul}(B-\nu)^p-\operatorname{nul}(B-\nu)^{p-1}\big).$$

(3.6) $\sigma_{em}(\boldsymbol{H})=$ the union of the right-hand side of (3.4) and the set of all $\lambda\in \boldsymbol{C}$ outside the right-hand side of (3.4) for which (3.5) does not vanish.

THEOREM 2. *Let α be a uniform reasonable norm on $X\otimes Y$ with the i-property. Then the following relations hold*:

$$\sigma_{\pi}(\boldsymbol{H}) = \sigma_{\pi}(A)+\sigma_{\pi}(B). \tag{3.7}$$

$$\sigma_{+}(\boldsymbol{H}) = \big(\sigma_{+}(A)+\sigma_{\pi}(B)\big)\cup\big(\sigma_{\pi}(A)+\sigma_{+}(B)\big). \tag{3.8}$$

If λ is outside the set (3.8), *then*

$$\operatorname{nul}(\boldsymbol{H}-\lambda) = \sum_{\substack{(\mu,\nu)\in\sigma_{\pi}(A)\times\sigma_{\pi}(B) \\ \mu+\nu=\lambda}} \sum_{p=1}^{\infty}\big(\operatorname{nul}(A-\mu)^p-\operatorname{nul}(A-\mu)^{p-1}\big)\times \tag{3.9}$$
$$\times\big(\operatorname{nul}(B-\nu)^p-\operatorname{nul}(B-\nu)^{p-1}\big).$$

THEOREM 3. *Let α be a uniform reasonable norm on $X\otimes Y$ with the h-property. Then the following relations hold*:

$$\sigma_{\delta}(\boldsymbol{H}) = \sigma_{\delta}(A)+\sigma_{\delta}(B). \tag{3.10}$$

$$\sigma_{-}(\boldsymbol{H}) = \big(\sigma_{-}(A)+\sigma_{\delta}(B)\big)\cup\big(\sigma_{\delta}(A)+\sigma_{-}(B)\big). \tag{3.11}$$

If λ is outside the set (3.11), *then*

$$\operatorname{def}(\boldsymbol{H}-\lambda) = \sum_{\substack{(\mu,\nu)\in\sigma_{\delta}(A)\times\sigma_{\delta}(B) \\ \mu+\nu=\lambda}} \sum_{p=1}^{\infty}\big(\operatorname{def}(A-\mu)^p-\operatorname{def}(A-\mu)^{p-1}\big)\times \tag{3.12}$$
$$\times\big(\operatorname{def}(B-\nu)^p-\operatorname{def}(B-\nu)^{p-1}\big).$$

(3.13) $\sigma_{ek}(\boldsymbol{H})=$ *the intersection of the right-hand side of* (3.8) *and the right-hand side of* (3.11).

Theorem 1 is proved by means of the Fredholm theory and perturbation theory by I. C. Gohberg–M. G. Krein and T. Kato (e.g. [13], [14]), and a reduction theorem of $\boldsymbol{H}$. The proof of Theorem 2 makes use of a result of Słodkowski–Żelazko [21] (cf. Choi–Davis [4]) on the joint approximate point spectrum of commuting bounded linear operators. Theorem 3 is shown by the dual argument of Theorem 2.

Remarks.

1. All the results can be extended to $\boldsymbol{P}$ with a certain class of polynomials $P(\xi,\eta)$ depending on A and B.

2. If α is a uniform reasonable norm on $X\otimes Y$ without the *i*- and *h*-property, Theorems 2 and 3 in general do not hold.

3. When X and Y are Hilbert spaces, Theorems 1–3 are true for the crossnorms π, ε, and the prehilbertian norm σ on $X\otimes Y$, more generally, for every $\otimes$-norm α. Note [12] that, in this case, $A\otimes I+I\otimes B$ is closable in $X\hat{\otimes}_{\alpha}Y$ with closure $\boldsymbol{H}$.

4. L. and K. Maurin [16] proved the relation (3.1) for both A and B self-adjoint operators (cf. Berezanskii [2]).

5. Reed–Simon [17] has used the Banach-algebra techniques to prove (3.1).

6. Schechter and Snow [19] treated the very special case of the relations (3.2) and (3.4) of Theorem 1 for both A and B bounded.

7. The relations (3.7) and (3.10) show that the spectral mapping theorems with the approximate point spectrum and approximate deficiency spectrum hold. But the point spectrum does not possess the spectral mapping theorem [11].

8. The relation (3.1) in Theorem 1 has been applied to the spectral theory of many-body Schrödinger operators with dilatation-analytic interactions by Balslev–Combes [1] and B. Simon [20].

9. The relations (3.4) and (3.6) in Theorem 1 may be used, together with the perturbation theorems in T. Kato [13], [14] and M. Schechter [18], to determine the Wolf and Schechter essential spectra of the perturbed operator $H=\boldsymbol{H}+V$ in $X\hat{\otimes}_{\alpha}Y$, when V is $\boldsymbol{H}$-compact.

10. Theorems 1–3 may contribute to the resolution of the linear operator equation $AU-UB=C$ in the Banach algebra of all bounded linear operators on a complex Banach space Z (cf. Davis–Rosenthal [5], R. E. Harte [8], Lumer–Rosenblum [15]).

11. The question is open how to represent the set

$$\{\lambda\in\boldsymbol{C};\ \boldsymbol{H}-\lambda \text{ does not have a closed range}\}.$$

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ON SPECTRAL DISTRIBUTIONS OF DEFINITIZABLE OPERATORS IN KREIN SPACE

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Let X be a Krein space, i.e. a Hilbert space with respect to some scalar product $(\cdot, \cdot)$, equipped with an indefinite scalar product $[\cdot, \cdot]$ given by $[x, y] = (Jx, y)$, $x, y \in X$. Here J denotes the difference of two orthogonal projectors P_+, P_- with $P_+ + P_- = I$ (the identity on X): $J = P_+ - P_-$. Let $\varkappa_\pm(X) := \dim P_\pm X \in \{0, 1, \ldots, \infty\}$. The quantities $\varkappa_\pm(X)$ are called the *rank of positivity* and the *rank of negativity*, respectively, of the Krein space X. A bounded operator A on X is said to be *J-self-adjoint* if $[Ax, y] = [x, Ay]$, $x, y \in X$. A J-self-adjoint bounded operator A on X is said to be *definitizable*, if there exists a real non-constant polynomial p with property $[p(A)x, x] \geqslant 0$, $x \in X$. The non-real spectrum of a definitizable operator can be proved ([6]) to consist of no more than a finite number of eigenvalues.

In what follows A denotes a bounded definitizable operator. We assume that the spectrum $\sigma(A)$ of A is real (for the following considerations this is, in fact, no restriction).

The spectral function $E(\cdot)$ of a definitizable operator was found by M. G. Krein and H. Langer ([5], [6]). It is a projector-valued interval function defined on all real intervals whose endpoints do not belong to the set $\{\mu_1, \ldots, \mu_k\}$ of real zeros of the definitizing polynomial p.

The Riesz–Dunford functional calculus $f \mapsto f(A)$ can be extended (cf. [6], [4]) to an $L(X)$-valued distribution which on $\boldsymbol{R} \setminus \{\mu_1, \ldots, \mu_k\}$ provides the measure corresponding to the interval function $E(\cdot)$. This distribution is also denoted by E. It is a spectral distribution in the sense of Foiaş (cf. [2]).

In the case of $\dim X < \infty$ (and for an arbitrary linear operator) the order of E in a neigbourhood of a point $\mu \in \sigma(A)$ is equal to the maximal length of Jordan chains in the root space of this point minus one.

In this note we are concerned with connections between the order of the distribution E on one-sided and deleted neighbourhoods of a point μ_i, $i = 1, \ldots, k$, and the length of Jordan chains in certain subspaces of the root space to μ_i.

M. G. Krein and H. Langer ([5], [6]) proved connections of the type considered here in the case of a Pontrjagin space, i.e. $\min(\varkappa_+(X), \varkappa_-(X)) < \infty$. Here