# SINGULARITIES SYMPOSIUM — ŁOJASIEWICZ 70

Editors of the Volume

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#### **PREFACE**

The present volume of the Banach Center Publications is dedicated to Stanisław Lojasiewicz on the occasion of his 70th birthday. It contains 26 papers concerning a wide spectrum of up-to-date topics of singularity theory; in particular, algebraic, semialgebraic, semianalytic and subanalytic geometry, o-minimal structures and singularities of differential equations.

The papers were written by the participants of two symposia which were held in Cracow and in Warsaw. The first was the Symposium on Singularities, on the Occasion of the 70th Birthday of Stanisław Lojasiewicz, held from 25 to 29 September 1996 at the Jagiellonian University, Cracow (Scientific Committee: Bernard Malgrange, Czesław Olech, Józef Siciak, René Thom and Jean-Claude Tougeron; Organizing Committee: Marek Jarnicki, Wiesław Pawłucki and Jacek Stasica). The symposium was continued during the Seminar on Singularities and Geometry, held from 30 September to 4 October 1996 at the Banach Center, Warsaw (Organizing Committee: Bronisław Jakubczyk, Stanisław Janeczko, Wiesław Pawłucki and Jacek Stasica). The full lists of lectures presented at both conferences are given on pp. 7–8.

The articles in the present volume are either original papers or extended versions of lectures delivered during the two symposia.

The editors wish to thank Dr. Jan Kowalski for his excellent technical and editorial preparation of the volume.

Bronisław Jakubczyk, Wiesław Pawłucki and Jacek Stasica

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#### LIST OF LECTURES

### Lectures presented at the Symposium on Singularities on the Occasion of the 70th Birthday of Stanisław Łojasiewicz

Cracow, September 25-29, 1996

- S. S. Abhyankar: Fundamental groups of algebraic varieties
- J. M. Aroca: Asymptotic development of functions in several variables
- E. Becker: The real holomorphy ring of a ring
- J.-P. Brasselet: About the relative de Rham theorem (a joint work with B. Teissier)
- L. Bröcker: On symmetric semialgebraic sets and orbit spaces
- F. Cano: Reduction of singularities of foliations and applications
- G. Choquet: Polish and French mathematics 50 years ago; a personal testimony
- M. Coste: Finiteness results in o-minimal structure
- M. Ferrarotti: Geodesic distance on stratified spaces
- J.-P. Françoise: Projection of analytic sets on polydiscs and Bernstein inequality
- M. Knebusch: Manis valuations and Prüfer extensions
- K. Kurdyka: The gradient conjecture of René Thom (a joint work with T. Mostowski)
- M. Lejeune-Jalabert: Arcs and wedges on a sandwiched surface singularity
- V. Sharko: On obstructions for topological equivalence of vector fields and functions on manifolds
- H. J. Sussmann: Optimal control applications of real analytic stratifications and desingularization
- B. Teissier: On the Lojasiewicz exponents
- A. Wilkie: A theorem of the complement for sets defined globally by smooth functions
- M.-A. Zurro: Gevrey rings

10 LECTURES

# Lectures presented at the Seminar on Singularities and Geometry

Warsaw, September 30-October 4, 1996

- S. S. Abhyankar: Polynomial expansion
- A. Agrachev: Exponential mappings in sub-Riemannian geometry
- F. Apéry: Pentagonal structure of the configuration space of five points in the real line
- J.-P. Brasselet: On a characterization of Gorenstein toric singularities
- J.-P. Françoise: Singularity theory and planar vector fields
- B. Jakubczyk and M. Zhitomirskii: Singularities of Pfaff equations
- Z. Jelonek: Testing sets for properness of polynomial mappings
- M. Knebusch: On valuation spectra
- M. Kwieciński: Finite sets in fibers of analytic maps
- J.-M. Lion: Theorems of Wilkie, van den Dries, Marker and Macintyre without Model Theory
- P. Mormul: Controllability by abnormal curves of a generic rank-3 distribution on a 5-manifold and its Cartan module
- T. Mostowski: The instability problem in classical mechanics
- A. Parusiński: Algebraically constructible functions and local topological properties of real algebraic sets
- F. Pelletier: Geometric properties of some 2-step sub-Riemannian manifolds
- W. Respondek: Feedback classification of nonlinear control systems: simple normal forms
- K. Spallek: Differential geometry on differentiable spaces and quotient spaces
- E. Strużyna and H. Zołądek: Analytic normal form of Bogdanov-Takens singularity
- ${\rm H.\ J.\ Sussmann:}\ An\ introduction\ to\ subanalyticity\ properties\ of\ sub-Riemannian\ balls\ and$   $other\ reachable\ sets$
- ${\bf Z.\ Szafraniec:}\ Algebraically\ constructible\ functions$
- V. Thilliez: Applications of Lojasiewicz inequalities to the study of very smooth functions
- A. Weber: Residues in intersection homology