TOPOLOGICAL ALGEBRAS, THEIR APPLICATIONS,
AND RELATED TOPICS
BANACH CENTER PUBLICATIONS, VOLUME 67
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 2005

WIESŁAW ŻELAZKO

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1. Life of Wiesław Żelazko. Wiesław Żelazko was born in Łódź, central Poland, on February 16, 1933, son of Zofia and Władysław. His father was an educator and social worker but the family roots can be traced to rural areas around Łódź and the iron smith tradition—hence the name Żelazko (iron in Polish). His early education was interrupted by the outbreak of the Second World War and the German occupation of Poland; he spent most of that period in Piotrków. In 1951 he graduated from high school and as one of the winners of the Second Mathematical Competition was admitted without entrance examination to the Warsaw University.

While the economic and political situation in Poland during that time was difficult, the scientific life, especially in mathematics, flourished. A large number of prominent mathematicians were active in Warsaw and other cities and enjoyed relative freedom of research. There were several weekly seminars on various topics; some, like the *Tuesday Functional Analysis Seminar*, continue till today. While the research activities in most other areas were severely restricted by the communist government, or by the lack of equipment, mathematics was perceived by the authorities as unrelated to the real world and consequently was often left alone.

Żelazko got his M.Sc. under the supervision of Prof. Roman Sikorski in 1955 and for the next two years worked as a teaching assistant for the Institute of Mathematics, Warsaw University. In 1957 he moved to the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) where he remains till today, for many years however maintaining close contacts with the Warsaw University. In fact these two institutions, while having different educational goals and obligations, often work as one large research group.

Żelazko obtained his doctorate in January of 1960 under the supervision of Prof. Stanisław Mazur. He become an adiunkt (1960), docent (1965), extraordinary professor

2000 *Mathematics Subject Classification*: Primary 01A70.
Key words and phrases: Wiesław Żelazko, topological algebras, Banach algebras.
The paper is in final form and no version of it will be published elsewhere.

(1971), and ordinary professor (1976). While working mostly in Warsaw he traveled extensively lecturing in over 40 countries and maintained mathematical contacts worldwide. His longer visits (6 months to a year) included Moscow State University, Yale University, Aarhus University, University of Kansas; he held shorter visiting positions at the University of Washington (Seattle), Université Paris-Sud, University of California Berkeley, University of Newcastle-upon-Tyne, Universidad de los Andes (Venezuela), Universidad National Autónoma de México, and many others.

Żelazko supervised well over 100 Master thesis and nine Ph.D. dissertations. The list of his doctorate students includes Piotr Uss, Czesław Matyszczyk, Zbigniew Słodkowski, Tomasz Müldner, Nguyen Van Khue, Jaroslav Zemánek, Ewa Ligocka, Andrzej Sołtysiak, and Krzysztof Jarosz. He also devoted a lot of time and energy to administrative and editorial work. For the last 35 years he has been an associate editor and later the Managing Editor of *Studia Mathematica*, he has been an associate editor of *Dissertationes Mathematicae*, member of the Editorial Committees of *Delta*, *Gradient*, and *Commentationes Mathematicae*; since 1972, for almost 20 years he worked as a deputy director of IMPAN.

2. Research of Wiesław Żelazko. The research of Wiesław Żelazko spans almost half a century and well over 100 papers, many of which started new lines of research. Here we will concentrate only on a few most important aspects of his research; any such selection is necessarily very subjective.

2.1. *Early papers.* The first paper by Żelazko ([1957a]) dealt with divisors of zero and the second one ([1957b]), coauthored with A. Białynicki-Birula, with multiplicative functionals. He returned to both topics several times in the following decades. In [1957b] he proved that in the Cartesian product $\prod_{t\in T} R_t$ of algebras R_t all multiplicative functionals are given in a trivial way, that is, as a composition of a projection on some R_{t_0} with a multiplicative functional on R_{t_0} if and only if the index set T is of nonmeasurable cardinality.

Other early papers leading to his Ph.D. in early 1960 concern primarily division algebras. One of the main theorems from this period states that a complete p-normed commutative division algebra over the field $\mathbb{C}$ of complex numbers is one-dimensional ([1960a]). The proof introduced new methods and did not use analytic function theory; indeed, the local compactness, connectivity and the fact that $\mathbb{C}$ is algebraically closed are the only properties of $\mathbb{C}$ used. That result is further generalized to show that a Hausdorff topological division algebra (only continuity of multiplication in each variable separately is assumed) over the complex numbers that has a bounded neighborhood of zero is one-dimensional. Many other basic theorems on normed and Banach algebras are consequently generalized to p-normed and complete p-normed algebras. Extending a well-known result on locally convex spaces Żelazko showed that a Hausdorff topological algebra whose topology is given by a family of submultiplicative p-seminorms (p variable) is topologically isomorphic to a subdirect product of a family of p-normed algebras. In the early sixties, by means of p-normed algebras Żelazko obtained generalizations of the classical Wiener and Wiener–Lévy theorems replacing the Fourier coefficients of class l_1 by coefficients of class $l_p, 0 < p \geq 1$. He also proved that for a locally compact group G

the space $L_p(G), 0 < p < 1$ is an algebra under convolution if and only if G is discrete; if G is commutative and $1 < p < \infty$, $L_p(G)$ is an algebra under convolution if and only if G is compact ([1961a], [1961b], see also [1963a], [1965b], [1975a]).

2.2. *Divisors of zero and generalized topological divisors of zero.* Żelazko's intense interest in divisors of zero and later generalized topological divisors started with his very first paper ([1957a]) written just after his M.Sc. degree. He proved there that any convolution Banach algebra $L_1(G)$ must contain divisors of zero.

The classical theorem of Shilov asserts that a real Banach algebra devoid of nontrivial topological divisors of zero is isomorphic to the field of real numbers, complex numbers, or quaternions. Seeking generalizations to arbitrary topological algebras Żelazko introduced in the sixties the concept of generalized topological divisors of zero and proved that an m-convex topological algebra without such divisors must be trivial, that is, equal to the field of complex numbers ([1966]). We say that a topological algebra $\mathcal{A}$ has *generalized topological divisors of zero* if there is a pair of subsets P, Q of $\mathcal{A}$ such that zero is in the closure of PQ, but neither in the closure of P nor in the closure of Q. Soon he extended the result showing that a real p-normed or m-convex algebra either contains generalized topological divisors of zero or is isomorphic to one of the three finite-dimensional real division algebras [1967a], [1967b]. It took almost two decades to show that the result is not true for B_0-algebras—in 1985 H. Arizmendi and W. Żelazko constructed a complex commutative B_0-algebra, not equal to $\mathbb{C}$, but without generalized topological divisors of zero [1985b]. The example is the set of power series $x = \sum_{i=0}^{\infty} \zeta_i t^i$ such that $\|x\|_n = \sum_{i=0}^{\infty} a_i^{(n)} |\zeta_i| < \infty$ $(n \in \mathbb{N})$, where $(a_i^{(n)}\colon\ i, n \in \mathbb{N})$ is a certain set of positive real numbers.

The most important paper concerning generalized topological divisors of zero was published in 1987 ([1987c]). Żelazko proved there that in any complete unital topological algebra without generalized topological divisors of zero the spectrum of any element, other than a multiple of identity, is empty or is equal to the entire scalar field. He also showed that any complex m-pseudoconvex algebra has generalized topological divisors of zero or is equal to the complex field. However the most significant result of that paper stated that for a real unital topological algebra $\mathcal{A}$ without generalized topological divisors of zero the group $G(\mathcal{A})$ of invertible elements must be isomorphic to one of three multiplicative groups: $\mathbb{R}\backslash\{0\}$, $\mathbb{C}\backslash\{0\}$, $\mathbb{H}\backslash\{0\}$. An important deduction that emerges from the above result is that in a complex topological algebra without generalized topological divisors of zero the group $G(\mathcal{A})$ contains only multiples of identity. The converse result is false even for commutative unital B_0 algebras [1987c]. For further extensions one may consult a survey article [1988a].

2.3. *Metric generalizations of Banach algebras.* In 1965 Żelazko published the first systematic survey of metric generalizations of Banach algebras, based on his lectures given at Yale University in 1963/64 ([1965a]). It contained many of his earlier results with some new theorems, some results of Arens, Michael, Williamson and others as well as several open problems. The first chapter contains mainly the theory of locally bounded complete metric algebras. It is shown that most important properties of Banach algebras are also true for p-normed algebras. The second chapter deals with F-algebras and topological division algebras, the third one with B_0-algebras, and the remaining part with the entire functions operating on topological algebras. This publication was a major part of his habilitation in 1965. A few years later, after a series of lectures at the University of Aarhus in 1969/70, the survey was revised, greatly enlarged and published in the Aarhus Lecture Notes Series ([1971c]).

2.4. *Entire functions operating on topological algebras.* If $\mathcal{A}$ is a complex Banach algebra and $\varphi(z) = \sum_{n=0}^{\infty} a_n z^n$ an entire function then φ operates on $\mathcal{A}$, that is, the series $\sum_{n=0}^{\infty} a_n x^n$ is convergent for any $x \in \mathcal{A}$. The statement is obvious and can also be easily proved for m-convex algebras. Clearly the submultiplicativity of the seminorms is essential in that simple proof but more surprisingly it is also necessary. In 1962 Żelazko together with B. Mitiagin and S. Rolewicz proved that if all entire functions operate on a given commutative B_0-algebra then it must be m-convex. On the other hand a single entire function is never enough for such a conclusion: for any entire function φ there exists a B_0-algebra R_φ such that φ operates on R_ϕ, but R_ϕ is not m-convex ([1962b]). It is again surprising that there is a noncommutative B_0-algebra on which all entire functions operate but which is not m-convex (of course all commutative subalgebras must be m-convex); such an example was however discovered much later ([1994c]). The subject of operating functions was also throughly discussed in [1965a] and [1971c].

2.5. *Gleason–Kahane–Żelazko Theorem.* 1968 was a particularly important year in the mathematical life of W. Żelazko; that year he published the proofs of now classical Gleason–Kahane–Żelazko Theorem ([1968a], [1968b]) and the Hirschfeld–Żelazko Theorem ([1968d]) as well as his influential book *Banach Algebras* ([1968e]).

THEOREM 1 (G-K-Ż). *If F is a linear functional on a complex unital Banach algebra $\mathcal{A}$ such that*

$$F(x) \neq 0 \quad \text{for } x \in \mathcal{A}^{-1}, \tag{1}$$

then $F/F(e)$ is multiplicative.

The theorem was proved in the commutative case independently, and surprisingly using the same method, by J.-P. Kahane and W. Żelazko ([1968a]) and by A. M. Gleason ([9]). The noncommutative case was settled in the same year by W. Żelazko ([1968b]). Since then the result has been phrased in many different ways, generalized in many directions, and influenced a large volume of research. It was immediately noticed that it can be easily generalized to maps between commutative Banach algebras:

THEOREM 2 (G-K-Ż). *If T is a linear map from a complex unital Banach algebra $\mathcal{A}$ into a complex unital commutative semisimple Banach algebra $\mathcal{B}$ such that $\sigma(T(x)) \subset \sigma(x)$ for $x \in \mathcal{A}$, then T is multiplicative.*

The obvious and important problem of how far this version of the Theorem could be extended to other algebras is still not completely settled in spite of many important discoveries, some of them very recent.

THEOREM 3 (B. Aupetit, [3]). *If T is a surjective linear map between complex unital semisimple von Neumann algebras such that $\sigma(T(x)) = \sigma(x)$ for all x, then T is a Jordan isomorphism.*

The result fails in general without the assumption that the algebras are semisimple or that T is surjective; it is not known if it holds for all C^*-algebras. A reader may consult survey articles [2] or [12] for more information and further references on this subject.

The Gleason–Kahane–Żelazko Theorem can also be phrased as follows.

THEOREM 4. *Let $\mathcal{A}$ be a complex unital Banach function algebra on a compact set X and let M be a codimension one subspace of $\mathcal{A}$. Then*

$$(\forall f \in M \ \exists x \in X \ \ f(x) = 0) \ \Rightarrow \ (\exists x \in X \ \forall f \in M \ \ f(x) = 0).$$

Almost immediately after the publication of the original paper by Kahane and Żelazko the question was raised if the assumption above that $\operatorname{codim} M = 1$ is essential. It is not very difficult to show that the theorem fails for general subspaces ([12]) but the question whether $\operatorname{codim} M < \infty$ is sufficient is still open after more than three decades. It is sufficient for $\mathcal{A} = C(X)$ and for some other algebras ([10]) and we do not know any complex Banach algebra that would fail that property.

Yet another line of research concerning generalizations of the Gleason–Kahane–Żelazko Theorem was initiated by R. Arens in 1987 ([4]). He asked whether the set $\mathcal{A}^{-1}$ in (1) can be replaced by a smaller set like $\varphi(\mathcal{A})$ for some entire function φ (an analysis of the original proof shows that $\mathcal{A}^{-1}$ can be replaced by $\exp(\mathcal{A})$). After a series of partial results by R. Arens ([4]), C. Badea ([5]), and R. Berntzen and A. Sołtysiak ([6]), the complete answer was obtained recently:

THEOREM 5 ([11]). *Let $\mathcal{A}$ be a complex Banach algebra with a unit e, let φ be a nonconstant entire function, and let F be a linear functional on $\mathcal{A}$. If the function $F \circ \varphi : \mathcal{A} \to \mathbb{C}$ is nonsurjective then $F = 0$ or $F/F(e)$ is multiplicative.*

2.6. *Hirschfeld–Żelazko Theorem.* One of the most important classes of Banach algebras are function algebras, also referred as uniform algebras. A function algebra $\mathcal{A}$ is a closed subalgebra of $C(X)$ equipped with the sup norm. Obviously

$$\|f\|^2 = \|f^2\| \quad \text{for } f \in \mathcal{A}, \tag{2}$$

and by the Gelfand Representation Theorem any commutative Banach algebra that satisfies (2) is isometrically isomorphic with a function algebra. Is the assumption that $\mathcal{A}$ is commutative superfluous? Hirschfeld and Żelazko gave an affirmative answer ([1968d]).

THEOREM 6. *Assume A is a complex Banach algebra such that*

$$\|f\|^2 \le c\|f^2\| \quad \textit{for } f \in \mathcal{A}, \tag{3}$$

for some positive constant c. Then $\mathcal{A}$ is commutative and consequently isomorphic with a function algebra.

This is not only a very elegant result but also one with numerous applications, generalizations and still related open problems. For example: is the result valid for real Banach algebras? Obviously not: we have a very simple counterexample, the noncommutative algebra $\mathbb{H}$ of quaternions. However this is only half of the answer; if a real Banach algebra satisfies (3) then it must be a (possibly noncommutative) function algebra, that is, a subalgebra of $C_{\mathbb{H}}(X)$ consisting of continuous $\mathbb{H}$-valued functions on a compact set X ([1]). What if the condition (3) is satisfied on each commutative subalgebra of $\mathcal{A}$ but possibly with different constants c for each subalgebra; must $\mathcal{A}$ be commutative? We do not know.

2.7. *Topologizable and nontopologizable algebras.* Typically in our area of study we investigate properties of a given Banach algebra, or a topological algebra, or of a class of such objects. Why not go deeper and ask how such objects could be created? Given just an algebra, can we always introduce a norm, or at least a topology to make it a topological algebra, preferably a nice one, like a locally convex algebra? Is such a topology unique? Such intriguing questions were considered for decades and Żelazko was attracted by them since the eighties while developing formal classification of topological algebras; [1992b] provides a list of interesting problems from that time. In 1990 he showed that the algebra $L(X)$ of all endomorphisms of a vector space X is topologizable as a locally convex topological algebra (with jointly continuous multiplication) if and only if it is topologizable as a Banach algebra, and this holds if and only if X is of finite dimension ([1990c]). That result was subsequently generalized by V. Müller ([13]) and others. In the following years Żelazko proved several further results on that subject. He showed that any real or complex countably generated algebra has a complete locally convex topology making it a topological algebra ([1994f]). Such a topology however may not be unique. Indeed, in a joint paper with M. Wojciechowski ([1997a]) they proved that the algebra $P(t)$ of all polynomials of one variable admits a continuum of different complete locally convex topologies. It has been known since the seventies ([7], [8]) that for a locally convex space X the algebra $B(X)$ of all continuous endomorphisms of X is topologizable only if X is subnormed. In one of the most recent papers ([2002b]) Żelazko showed that when X is sequentially complete this condition is also sufficient and obtained some other conditions equivalent to topologizability of $B(X)$.

2.8. *From subalgebras to the algebra.* If all commutative subalgebras of a Banach algebra $\mathcal{A}$ are isometric with a uniform algebra then $\mathcal{A}$ is commutative and also a uniform algebra. If a linear functional F on $\mathcal{A}$ is multiplicative on any commutative subalgebra, then F is multiplicative. There are several results like these that allow us to make conclusions about the entire algebra based on properties of nice subalgebras. Żelazko was very interested in such problems for many years, in fact the two statements just mentioned follow from the Hirschfeld–Żelazko Theorem and from the Gleason–Kahane–Żelazko Theorem, respectively. A closely related question asks when an algebra could be generated by a small number of much simpler subalgebras.

In general, for a topological algebra such questions often have a negative answer. For example: there is a semitopological but non-topological algebra (that is, an algebra with separately but not jointly continuous multiplication) with every commutative subalgebra topological ([1997a]), also there is a non-locally convex topological algebra with all commutative subalgebras locally convex ([1996e]).

The problem whether an algebra $\mathcal{A}$ can be generated by a few commutative subalgebras is particularly interesting for $\mathcal{A} = B(X)$, the algebra of all continuous endomorphisms of a Banach space X. Żelazko showed that $B(X)$ is algebraically generated by two of its commutative subalgebras $\mathcal{A}_1$ and $\mathcal{A}_2$ of square zero whenever $X = X_1 \oplus \cdots \oplus X_n$, where $X_1, \ldots, X_n$ are mutually isomorphic, but $B(X)$ may not be even topologically generated by such two algebras in general ([1990a]). On the other hand there always exist two subalgebras of square zero which topologically generate $B(X)$ with respect to the strong operator topology ([1988b]). If X is separable $B(X)$ can even be topologically generated in strong operator topology by just two elements of $B(X)$ ([1989a]). Subsequently Żelazko proved that many of such conditions are equivalent, for example the following three: (i) X is a square, that is, X decomposes into a direct sum of two closed subspaces $X = X_1 \oplus X_2$ where X_1 and X_2 are isomorphic; (ii) $B(X)$ is algebraically generated by two subalgebras of square zero, one of them being of dimension one ([1988b]), and (iii) $B(X)$ is uniformly generated by two subalgebras of square zero, one of them being of dimension one ([1992a]).

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