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Correction to the paper 'Copies of ℓ_∞ in the space of Pettis integrable functions with integrals of finite variation'

(Studia Math. 210 (2012), 93–98)

by

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If X is a Banach space over the field $\mathbb{K}$ of real or complex numbers and (Ω, Σ, μ) is a complete probability space, it is asserted in [1, Lemma 2.1] that (under the assumption that $X \not\supset \ell_\infty$) if the linear subspace $\mathcal{M}(\Sigma, \mu, X)$ of $\mathcal{P}_1(\mu, X)$ consisting of those functions $f : \Omega \to X$ with indefinite Pettis integrals of bounded variation, equipped with the variation norm $|f|_\Sigma$, has a copy of ℓ_∞ then there exists a countably generated sub-σ-algebra Γ of Σ and a closed and separable linear subspace Y of X such that $\mathcal{M}(\Gamma, \mu|_\Gamma, Y)$ contains an isomorphic copy of ℓ_∞. If K is an isomorphism from ℓ_∞ into $\mathcal{M}(\Sigma, \mu, X)$ and $J := SK$, where $Sf(E)$ stands for the Pettis integral of f on $E \in \Sigma$, once Y and Γ have been constructed in the paper and has been shown that one may assume that $J\xi(E) \in Y$ for all $\xi \in \ell_\infty$ and $E \in \Gamma$, it turns out that the map $T : \ell_\infty \to \mathcal{M}(\Gamma, \mu|_\Gamma, Y)$ intending to carry a copy of ℓ_∞ into $\mathcal{M}(\Gamma, \mu|_\Gamma, Y)$ is not well-defined. The definition requires $T\xi$ to be weakly $\mu|_\Gamma$-measurable and Y-valued. A sufficient condition to get this is to require Y to have the $\mu|_\Gamma$-weak Radon–Nikodým property (WRNP). In this case, since $J\xi|_\Gamma : \Gamma \to Y$ is a $\mu|_\Gamma$-continuous measure of finite variation there is $h_\xi \in \mathcal{P}_1(\mu|_\Gamma, Y)$ such that

$$J\xi(E) = (P) \int_E h_\xi(\omega)\, d\mu|_\Gamma(\omega)$$

for each $E \in \Gamma$, so that the operator $T : c_0 \to \mathcal{M}(\Gamma, \mu|_\Gamma, Y)$ given by $T\xi = h_\xi$ is well-defined, linear and bounded. Since $|Te_n|_\Gamma = |Ke_n|_\Sigma$ for all $n \in \mathbb{N}$, as shown in the paper, Rosenthal's ℓ_∞ theorem yields $\mathcal{M}(\Gamma, \mu|_\Gamma, Y) \supset \ell_\infty$. So, the main result [1, Theorem 2.2] must be restated as follows.

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Theorem. *If each closed and separable subspace Y of X has the $\mu|_{\Gamma}$-WRNP for each sub-σ-algebra Γ of Σ, then $\mathcal{M}(\Sigma, \mu, X)$ contains a copy of ℓ_∞ if and only if X does.*

References

[1] J. C. Ferrando, *Copies of ℓ_∞ in the space of Pettis integrable functions with integrals of finite variation*, Studia Math. 210 (2012), 93–98.

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