MATHEMATICAL PROBLEMS IN COMPUTATION THEORY
BANACH CENTER PUBLICATIONS, VOLUME 21
PWN - POLISH SCIENTIFIC PUBLISHERS
WARSAW 1988

FINITE CATEGORIES AND REGULAR LANGUAGES

DENIS THÉRIEN
MAŁGORZATA SZNAJDER-GLODOWSKI

School of Computer Science, McGill University, Montréal, Québec, Canada

The traditional approach in algebraic theory of automata is to represent finite-state machines as finite monoids. Several recent results have been obtained by considering categories instead of monoids. In this article, we will argue that this generalization is very natural, that it preserves many important classical results, and that it allows new methods for solving old problems.

Introduction

Traditionally (e.g. [2]), one looks at a finite-state machine as processing sequences of inputs drawn from a finite set A, the input alphabet: one then considers the free monoid A^*, i.e., the set of all sequences of finite length over A, as the input universe to the machine. To the automaton can be associated a congruence γ of a finite index on A^*. A^*/γ being a finite monoid, one is then led to investigate relationships between the structure of this algebraic system and the combinatorial processing of input sequences.

Recent research has established the possibility and the necessity of generalizing this model. For example, we are often interested in decompositions of automata. In such situations a component may receive its input from the output of some other component. This "preprocessing" imposes restrictions on the possible input sequences that need to be considered. A simple way to take into account these restrictions is to view a machine as processing input sequences that are paths in a finite directed multigraph. The input universe is then the free category induced by the graph and we can associate to the machine a congruence of finite index on this free category. The machine thus becomes a finite category rather than a monoid. Note that a free monoid is simply a free category induced by a one-vertex graph.

Thus the categorical model allows us to better formalize the description of a machine when we are dealing with decomposition problems. In this paper we will show that it is quite possible to reformulate classical notions in the new setting. In particular, we will discuss regular and star-free languages, varieties of categories and wreath product operation, and we will generalize the monoid-version theorems about these concepts. Hence it is possible to recover virtually all of the original theory in the new frame-work. Finally, we will indicate how the extension has provided new methods for solving old problems.

1. Notation

Let (N, A) be a finite directed multigraph where N is the set of vertices and A the set of edges. For $i, j \in N$ we denote by A^*_{ij} the set of all paths of finite length from i to j, including if $i = j$ a path of length 0 denoted 1_i. Let $A^* = \bigcup_{i,j \in N} A^*_{ij}$ represent the set of all paths of finite length in the multigraph: concatenation of consecutive paths is associative, i.e., for all $i, j, k \in N$, $s \in A^*_{ij}$, $t \in A^*_{jk}$, $u \in A^*_{kl}$, $(st)u = s(tu)$ and the 0-length paths act as identities, i.e., for all $s \in A_{ij}$, $t \in A_{jk}$, $s 1_j = s$ and $1_j t = t$. A^* is the *free category* induced by the multigraph (N, A). A *language* is any subset of A^*.

A *congruence* γ on A^* is an equivalence relation satisfying

(1) $[x]_\gamma \subseteq A^*_{ij}$,

(2) $[x]_\gamma [y]_\gamma \subseteq [xy]_\gamma$.

For example, the coarsest congruence on A^* is defined by $x \sim y$ iff $x, y \in A^*_{ij}$.

We say that $L \subseteq A^*$ is a *γ-language* if L is a union of classes of the congruence γ. For any language L, the *syntactic congruence* of L is defined by $x \ \gamma_L \ y$ iff $x \sim y$ and for all $u, v \in A^*$ such that uxv, uyv are defined, we have $uxv \in L$ iff $uyv \in L$. It is clear that L is a γ_L-language and that for any congruence γ on A^*, L is a γ-language iff $\gamma \subseteq \gamma_L$. Observe that a free monoid is simply a free category induced by a one-object graph and that in this case all notions introduced above reduce to their usual version.

The *index* of a congruence is the number of classes. A congruence γ is *aperiodic* if for all $i \in N$, for all $x \in A^*_{ii}$ there exists $t \geq 0$ such that $x^t \ \gamma \ x^{t+1}$.

Languages being subsets of A^*, one can consider boolean operations on languages in the usual way. Given $L, L_1, L_2 \in A^*$, $u, v \in A^*$ we also consider the following operations: the *quotient* $u^{-1} L v^{-1}$ is defined as $\{x | \ uxv \in L\}$, the *concatenation* $L_1 L_2$ is defined as $\{xy | \ x \in L_1, y \in L_2$ and xy is a defined path$\}$, the *nth power* of L $(n > 0)$ is defined as $L^1 = L$, $L^n = L^{n-1} L$, and the *positive iteration* L^+ is defined as $\bigcup_{n>0} L^n$.

Any category C can be *represented* as the quotient of a free category by a congruence. If C is on the set of objects N and has a set of arrows generated by A, then $C = A^*/\gamma$, where A^* is the free category induced by (N, A) and $x\gamma y$ iff the sequences of arrows x and y multiply to the same value in C. This representation is in general not unique, since the set of generators can be chosen in different ways.

Let C and D be categories on objects N and M respectively. A *relational morphism* φ: $C \to D$ is given by a mapping φ: $N \to M$ and a relation φ between arrows of C and arrows of D such that $s\varphi \neq \emptyset$ for any arrow s of C, $C_{ij}\varphi \subseteq D_{i\varphi j\varphi}$ where C_{ij} is the set of arrows of C from object i to j, $1_{i\varphi} \in 1_i\varphi$ and $(s\varphi)(t\varphi) \subseteq (st)\varphi$. We say that C divides D, noted $C \prec D$, if there exists a relational morphism φ: $C \to D$ such that for any $s, t \in C_{ij}$, $s\varphi \cap t\varphi \neq \emptyset$ implies $s = t$. If C and D are monoids (i.e. one-object categories) it can be shown that $C \prec D$ iff C is a morphic image of a submonoid of D. A *functional morphism* is a relational morphism in which the arrow relation is a function. The *direct product* $C \times D$ is the category with objects $N \times M$ and set of arrows $\{\{s, t\} \mid s$ is an arrow of C, t is an arrow of $D\}$.

2. Three theorems

In this section we will prove three fundamental theorems of the classical theory of automata in the extended categorical setting. These are Kleene's theorem on regular languages [4], Schützenberger's theorem on star-free languages [6] and Eilenberg's theorem on varieties [1].

Let A^* be the free category induced by the graph (N, A). The class of *star-free languages* over A^* is the smallest family of subsets of A^* that contains $\emptyset$, $\{1_i\}$ for each $i \in N$, $\{a\}$ for each $a \in A$, and that is closed under boolean operations and concatenation. The class of *regular languages* (over A^*) is the smallest family of subsets of A^* containing the star-free languages and that is closed under positive iteration.

Kleene's theorem asserts that for one-object free categories (i.e. free monoids) a language L is regular iff there exists a finite-index congruence γ such that L is a γ-language. Schützenberger's theorem says that, for the one-object case, a language L is star-free iff there exists a finite-index aperiodic congruence γ such that L is a γ-language. In both these theorems, if the empty word is not in L the congruence can be taken over A^+.

The extended versions of these results can be moved by reducing the general case to the one-object situation. The technicality is to handle correctly the paths of length 0. To the free category A^* we will associate the free semigroup A_s^+ generated by A, disregarding the fact that elements of A are edges in a graph: thus any non-empty finite sequence of elements of A belongs to A_s^+, regardless of whether it is a valid path or not. If γ_s is a congruence on A_s^+, we denote by γ the congruence on A^* defined by $x\gamma y$ iff

$x = y = 1_i$ or $|x| > 0$, $|y| > 0$, $x \sim y$ and $x\gamma_s y$ as elements of A_s^+. For any $L \subseteq A^*$, we define $L_s = \{x|\ x \in L,\ |x| > 0\}$; L_s is viewed as a subset of A_s^+.

LEMMA 2.1. *If $L \subseteq A^*$ is regular (star-free), then L_s is regular (star-free).*

The result is clear for $\emptyset$, $\{1_i\}$ and $\{a\}$. To carry the induction step, simply observe that $(L \cup L')_s = (L_s \cup L'_s)$, $(LL')_s = L_s L'_s$ and $(L^+)_s = (L_s)^+$. ∎

LEMMA 2.2. *If γ_s is a finite-index (aperiodic) congruence on A_s^+, then so is γ over A^*.*

We have $|A^*/\gamma| \leqslant |N| + |N \times N|\,|A_s^+/\gamma_s|$. Also if $x = 1_i$ then $x\gamma x^2$ and if $x \in A_{ii}^*$, $|x| > 0$ then $x^t \gamma_s x^{t+1}$ implies $x^t \gamma x^{t+1}$. ∎

THEOREM 2.3. *$L \subseteq A^*$ is regular iff there exists a finite-index congruence γ on A^* such that L is a γ-language.*

If L is regular as a subset of A^*, then L_s is also regular as a subset of A_s^+. By Kleene's theorem there exists a finite-index congruence γ_s on A_s^+ such that L_s is a γ_s-language. Then L is a γ-language and by Lemma 2.2 γ has finite-index. Conversely if L is a γ-language, then we may define β_s on A_s^+ by $x\beta_s y$ iff $x, y \in A^+$ and $x\gamma y$ or $x, y \notin A^+$: β_s has finite-index, L_s is a β_s-language, hence L_s is regular. Clearly L is also regular. ∎

THEOREM 2.4. *$L \subseteq A^*$ is star-free iff there exists a finite-index aperiodic congruence γ on A^* such that L is a γ-language.*

The proof is similar to that of Theorem 2.3, with the observation that if γ is aperiodic then the constructed β_s is also aperiodic. ∎

In the monoid case, a most powerful tool to relate combinatorial descriptions of languages to the algebraic properties of their syntactic recognizers is the notion of a variety. A collection V of finite monoids is a *M-variety* iff it is closed under direct product ($M_1, M_2 \in V$ implies $M_1 \times M_2 \in V$), under submonoids ($M_1 \in V$ and $M_2 \subseteq M_1$ implies $M_2 \in V$) and under morphic images ($M_1 \in V$ and $M_2 = M_1\varphi$ for some morphism φ implies $M_2 \in V$). Next, let $A^* V$ be a collection of regular languages over the free monoid A^* and let $V = \bigcup A^* V$, where the union is over all finite sets A: V is a *$*$-variety* iff $A^* V$ is a boolean algebra closed under left and right quotients ($L \in A^* V$, $u, v \in A^*$ implies $u^{-1} L v^{-1} \in A^* V$) and V is closed under inverse functional morphisms ($L \in A^* V$, φ: $B^* \to A^*$ a functional morphism implies $L\varphi^{-1} \in B^* V$). Eilenberg's theorem states that there exists a natural 1-1 correspondence between M-varieties and $*$-varieties.

These notions have natural generalizations. A *C-variety* is a collection of finite categories closed under division and direct product. The correct notion of $*$-variety is obtained from the definition above by replacing free monoids by free categories. Let V be any collection of finite categories: we write $V \Rightarrow V$ if $A^* V = \{L|$ there exists γ such that L is a γ-language and $A^*/\gamma \in V\}$. If V is any collection of regular languages, then we write $V \Rightarrow V$ if $V = \{A^*/\gamma|$ every γ-language belongs to $A^* V\}$. The argument for the generalized version of

Eilenberg's theorem will use the following lemmas, whose proofs are left to the reader.

LEMMA 2.5. *Let* $C = B^*/\beta$, $D = A^*/\gamma$. *Then* $C \prec D$ *iff there exists a functional morphism* φ: $B^* \to A^*$ *such that* $\beta \supseteq \varphi\gamma$.

LEMMA 2.6. (a) $A^*/\gamma_1 \cap \gamma_2 \prec A^*/\gamma_1 \times A^*/\gamma_2$,

(b) *for any functional morphism* φ: $B^* \to A^*$, $L \subseteq A^*$, *we have* $B^*/\gamma_{L\varphi^{-1}} \prec A^*/\gamma_L$.

LEMMA 2.7. *If* V *is a C-variety, then* $A^*/\gamma \in V$ *iff* $A^*/\gamma_L \in V$ *for all* γ-*languages* L.

LEMMA 2.8. *Let* V *be a C-variety. Then* $C \in V$ *iff any congruence representation of* C *belongs to* V.

THEOREM 2.9. (a) *If* V *is a C-variety and* $V \Rightarrow \mathcal{V}$, *then* $\mathcal{V}$ *is a* $*$-*variety and* $\mathcal{V} \Rightarrow V$.

(b) *If* $\mathcal{V}$ *is a* $*$-*variety and* $\mathcal{V} \Rightarrow V$, *then* V *is a C-variety and* $V \Rightarrow \mathcal{V}$.

(a) Let L, L_1, $L_2 \subseteq A^* \mathcal{V}$ and let γ, γ_1 and γ_2 be their respective syntactic congruences. Then $L_1 \cup L_2$ is a $(\gamma_1 \cap \gamma_2)$-language and it belongs to $A^* \mathcal{V}$ by Lemma 2.6(a); also $\bar{L}$ is a γ-language so that $A^* \mathcal{V}$ is a boolean algebra. For any $u, v \in A^*$, $u^{-1} L v^{-1}$ is a γ-language; hence $A^* \mathcal{V}$ is closed under quotients. Let φ: $B^* \to A^*$ be a functional morphism: $L\varphi^{-1}$ is $\varphi\gamma$-language and we get that $\mathcal{V}$ is closed under inverse morphisms by Lemma 2.6(b). Thus $\mathcal{V}$ is a $*$-variety. Suppose now that $\mathcal{V} \Rightarrow V'$ and let $C = A^*/\gamma$. If $C \in V'$ then all γ languages belong to $A^* \mathcal{V}$, which implies that all syntactic congruences γ_L, where L is any γ-language, are such that $A^*/\gamma_L \in V$: thus $C \in V$ by Lemma 2.7 and $V' \subseteq V$. If $C \in V$ then all γ-languages are in $A^* \mathcal{V}$ and $C \in V'$: this proves $V \subseteq V'$ and $V = V'$.

(b) Suppose $D = A^*/\gamma \in V$ and $C = B^*/\beta \prec D$. Each β-language is a $\varphi\gamma$-language by Lemma 2.5 for some functional morphism φ: $B^* \to A^*$. Since any $\varphi\gamma$-class is the inverse image by φ of a γ-language, we deduce that $C \in V$, i.e., V is closed under division. Let now $C_1, C_2 \in V$ and consider any congruence representation B^*/β of $C_1 \times C_2$. By Lemma 2.8, it suffices to argue that B^*/β is in V. We can construct B_1^*, B_2^*, β_1 and β_2 such that $C_1 = B_1^*/\beta_1$, $C_2 = B_2^*/\beta_2$. Let φ_1, φ_2 be the natural projections: for any $x \in B^*$ we can then find $L_1 \subseteq B_1^*$, $L_2 \subseteq B_2^*$ such that $[x]_\beta = L_1 \varphi_1^{-1} \cap L_2 \varphi_2^{-1}$, where L_i is a β_i-language for $i = 1, 2$. This established that any β-class, and hence any β-language belongs to $\mathcal{V}$, so that V is closed under direct product. Finally suppose that $V \Rightarrow \mathcal{V}'$. If $L \in A^* \mathcal{V}'$ then $A^*/\gamma_L \in V$ and all γ_L-languages belong to $A^* \mathcal{V}$: this proves $\mathcal{V}' \subseteq \mathcal{V}$. If $L \in A^* \mathcal{V}$ then $A^*/\gamma_L \in V$ since for any $x \in A^*$ we have $[x]_{\gamma_L} = \bigcap_{uxv \in L} u^{-1} L v^{-1} \cap \bigcap_{uxv \notin L} \overline{u^{-1} L v^{-1}}$ (note that the number of different terms in the boolean function is finite since L is regular). It follows that $\mathcal{V} \subseteq \mathcal{V}'$ and the proof is complete. ∎

3. Wreath product of C-varieties

A natural operation to consider on automata is the series (or cascade) connection, where the output line of a first machine is hooked up to the input line of a second machine. This physical connection has an exact algebraic description: the wreath product $S \circ T$ of two monoids S and T is the monoid defined as the set $S^T \times T$, where S^T is the set of all functions from T to S, with the operation $(f_1, t_1)(f_2, t_2) = (f, t_1 t_2)$, where f is the function defined by $tf = tf_1 (tt_1) f_2$. Given two M-varieties V and W, we denote by $V * W$ the M-variety $\{M \mid M \prec S \circ T$ for some $S \in V$ and $T \in W\}$. Many interesting decomposition results on varieties are based on this construction.

To define a similar operation for C-varieties it is convenient once more to use the congruence point of view. Let A^*/γ be a finite category; we form a new directed multigraph (N_γ, A_γ), where $N_\gamma = A^*/\gamma$ and $A_\gamma = \{([x]_\gamma, a) \mid a \in A,\ x, xa \in A^*\}$: the edge $([x]_\gamma, a)$ goes from the vertex $[x]_\gamma$ to the vertex $[xa]_\gamma$. Note that all paths in $[x]_\gamma$ being coterminal, A_γ is well defined. For any $x, y \in A^*$ such that $xy \in A^*$, there exists a path denoted ${}_x\bar{y}$ in the free category A_γ^* which is defined as follows: if $|y| = 0$ then ${}_x\bar{y} = 1_{[x]_\gamma}$, the identity path on the vertex $[x]_\gamma$: if $y = za$ then ${}_x\bar{y} = {}_x\bar{z}([xz]_\gamma, a)$.

The following facts are obvious:

(i) There is a 1-1 correspondence between A_γ^* and $\{{}_x\bar{y} \mid xy \in A^*\}$,

(ii) ${}_u\bar{v} \sim {}_x\bar{y}$ iff $u\gamma x$ and $uv\gamma xy$,

(iii) $|{}_x\bar{y}| = |y|$,

(iv) ${}_x\overline{yz} = {}_x\bar{y}\,{}_{xy}\bar{z}$.

Let β be a finite-index congruence on A_γ^*. We can now define an equivalence relation $\beta * \gamma$ on A^* by $x \beta * \gamma y$ iff $x\gamma y$ and for all $u \in A^*$ such that $ux \in A^*$ ${}_u\bar{x} \beta\, {}_u\bar{y}$.

LEMMA 3.1. *$\beta * \gamma$ is a finite-index congruence on A^*.*

If $x \beta * \gamma y$ then $x\gamma y$ so that we must have $x \sim y$: also note that we then have ${}_u\bar{x} \sim {}_u\bar{y}$ and the definition makes sense. Suppose now that $x \beta * \gamma y$ and $v \beta * \gamma w$. Then $xv\gamma yw$ since γ is a congruence. Moreover, ${}_u\overline{xv} = {}_u\bar{x}\,{}_{ux}\bar{v}$ and ${}_u\overline{yw} = {}_u\bar{y}\,{}_{uy}\bar{w}$. Since $x\gamma y$ we know that $ux\gamma uy$: we also have ${}_u\bar{x} \beta\, {}_u\bar{y}$ and ${}_{ux}\bar{v} \beta\, {}_{uy}\bar{w}$ so that ${}_u\overline{xv} \beta\, {}_u\overline{yw}$, and $\beta * \gamma$ is indeed a congruence. Finally the index of $\beta * \gamma$ is bounded by $|A^*/\gamma|^2 |A_\gamma^*/\beta|$. ∎

If β is a congruence on A_γ^* we will say that β is basic if ${}_{1_i}\bar{x} \beta\, {}_{1_i}\bar{y}$ (where x, y are assumed to start at vertex i) implies ${}_u\bar{x} \beta\, {}_u\bar{y}$ for all u such that $ux \in A^*$. Thus, when β is basic, $x \beta * \gamma y$ iff $x\gamma y$ and ${}_{1_i}\bar{x} \beta\, {}_{1_i}\bar{y}$.

LEMMA 3.2. *Let V be a C-variety such that $A_\gamma^*/\beta \in V$. Then there exists β' such that $A_\gamma^*/\beta' \in V$, β' is basic and $\beta' \subseteq \beta$.*

Define on A_γ^* ${}_u\bar{x} \beta'\, {}_u\bar{y}$ iff ${}_{wu}\bar{x} \beta\, {}_{wu}\bar{y}$ for all w such that $wu \in A^*$. It is clear that β' is a basic congruence and that $\beta' \subseteq \beta$. For any $w \in A^*$ define Ψ_w: A_γ^*

$\to A^*_\gamma$ by

$$[u]_\gamma \Psi_w = \begin{cases} [wu]_\gamma & \text{if } wu \in A^*, \\ [u]_\gamma & \text{otherwise,} \end{cases}$$

$$_u\bar{x}\Psi_w = \begin{cases} _{wu}\bar{x} & \text{if } wu \in A^*, \\ _u\bar{x} & \text{otherwise.} \end{cases}$$

It can be verified that Ψ_w is a well-defined morphism. Since Ψ_w is identical to Ψ_v wherever $w\gamma v$, there are only finitely many such morphisms: denote them by $\Psi_1, \ldots, \Psi_r$. We then get $\beta' \supseteq \Psi_1\beta \cap \ldots \cap \Psi_r\beta$ and this implies that $A^*_\gamma/\beta \in V$. ∎

Let now V and W be C-varieties. Define $V * W = \{A^*/\delta \mid \delta \supseteq \beta * \gamma$ where $A^*/\gamma \in W$ and $A^*_\gamma/\beta \in W\}$.

THEOREM 3.3. *$V * W$ is a C-variety.*

If $A^*/\delta_1 \in V * W$ and $\delta_2 \supseteq \delta_1$ then by definition $A^*/\delta_2 \in V * W$. Let now $\delta_1 \supseteq \beta_1 * \gamma_1$ and $\delta_2 \supseteq \beta_2 * \gamma_2$: it is direct to check that $\delta_1 \cap \delta_2 \supseteq (\varphi_1\beta_1 \cap \varphi_2\beta_2) * (\gamma_1 \cap \gamma_2)$ where φ_i: $A_{\gamma_1 \cap \gamma_2} \to A^*_{\gamma_i}$ is defined by $[u]_{\gamma_1 \cap \gamma_2}\varphi_i = [u]_{\gamma_i}$ and $_u\bar{x}\varphi_i = {}_u\bar{x}$ (note that the occurrence of $_u\bar{x}$ on the left-hand side is over $A^*_{\gamma_1 \cap \gamma_2}$ whereas the one on the right-hand side is over $A^*_{\gamma_i}$). Hence $A^*/\delta_1 \cap \delta_2 \in V$. Finally let $\delta \supseteq \beta * \gamma$ and φ: $B^* \to A^*$. We claim that $\varphi\delta \supseteq \Psi\beta * \varphi\gamma$ where Ψ: $B^*_{\varphi\gamma} \to A^*_\gamma$ is defined by $[u]_{\varphi\gamma}\Psi = [u\varphi]_\gamma$ and $_u\bar{x}\Psi = {}_{u\varphi}\bar{x}\bar{\varphi}$. Indeed let x $\Psi\beta * \varphi\gamma y$: then $x\varphi\gamma y$ so $x\varphi\gamma y\varphi$. Also $_{l_i}\bar{x}\,\Psi\beta\,{}_{l_i}\bar{y}$ implies $_{l_i\varphi}\bar{x}\,\bar{\varphi}\beta\,{}_{l_i\varphi}\bar{y}\bar{\varphi}$. We can assume that β is basic by Lemma 3.2: hence $_v\bar{x}\bar{\varphi}\beta\,{}_v\bar{y}\bar{\varphi}$ for any $v \in A^*$ such that $v(x)\varphi \in A^*$. This shows that $x\varphi\beta * \gamma y\varphi$ so that $x\varphi\delta y\varphi$ and $x\varphi\delta y$. Hence $B^*/\varphi\delta \in V * W$. ∎

We close this section by emphasizing the fact that this definition of wreath product of varieties reduces to the traditional one when only one-object categories are used. Also it enjoys all the properties that one would expect from his (her) one-object experience, e.g. it is more powerful than the join, i.e., $V \vee W \subseteq V * W \cap W * V$, where $V \vee W = \{C \mid C \prec C_1 \times C_2,\ C_1 \in V,\ C_2 \in W\}$, it is associative, i.e., $(U * V) * W = U * (V * W)$, etc.

4. Conclusion

We have shown that the theory of regular languages could be naturally extended to categories. This generalization allows one to recover many important results. In this conclusion, we would like to argue that this extension is not only possible, but also necessary.

The motivation for going to categories came from the right understanding of the wreath product. In Section 3, we have indicated how to construct the wreath product of two categories by introducing a free category A^*_γ which is obtained from $C = A^*/\gamma$. One should note that even when C is a monoid, this intermediate free system A^*_γ will have more than one object. Indeed, given monoids S and T, the "best" possible solution to the equation $S \prec X \circ T$ is a category ([7], [8]). Several new results on series decompositions of automata are based on this observation; for example, an effective characterization of dot-depth one languages was obtained by Knast [3] by making crucial use of categorical notions.

It can also be shown that the classification of categories in terms of varieties is finer than the corresponding one for monoids. There are C-varieties with no corresponding M-varieties. We thus have more building blocks to express decomposition theorems. For example, [5] studies families of languages that are defined by unambiguous concatenation: it is shown that this language operation can be algebraically understood in terms of the C-variety of so-called locally trivial categories. In monoid land this result is impossible to express since this C-variety has no one-object interpretation.

References

[1] S. Eilenberg, *Automata. Languages and Machines*, vol. B., Academic Press, New York 1976.

[2] A. Ginzburg, *Algebraic Theory of Automata*, Academic Press, New York 1968.

[3] R. Knast, *A semigroup characterization of dot-depth one languages*, RAIRO Informatique Théorique (1984).

[4] S. C. Kleene, *Representation of events in nerve nets and finite automata*, Automata Studies (C. E. Shannon and J. McCarthy, ed.), 3–42, Princeton University Press, Princeton, New-Jersey, 1956.

[5] J. E. Pin, H. Straubing and D. Thérien, *Locally trivial categories and unambiguous concatenation*, submitted for publication (1985).

[6] M. P. Schützenberger, *On finite monoids having only trivial subgroups*, Information and Control 8 (1965), 190–194.

[7] D. Thérien and A. Weiss, *Graph congruences and wreath products*, J. P. Ap. Alg. 36 (1985), 205–215.

[8] B. Tilson, *Categories as algebras, an essential ingredient in the theory of semigroups*, in preparation.

Presented to the semester
Mathematical Problems in Computation Theory
September 16–December 14, 1985