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A NON CHEBYSHEV FINITE DIMENSIONAL SUBSPACE IN H_1

S. B. STECHKIN

Steklov Mathematical Institute, Academy of Sciences of the U.S.S.R.
Moscow, U.S.S.R.

It is well known that finite-dimensional subspaces in L_p, $1 < p < \infty$, are Chebyshev subspaces. In the L_1 case such statement is simply not true. In some respects the classical Hardy H_1 space is more natural extension of the L_p spaces than the L_1 space. This note answers the following question asked by Z. Ciesielski: are all the finite-dimensional subspaces in the Hardy space H_1 Chebyshev? The answer is no.

Defining

$$\|f\|_{H_1} = \frac{1}{2\pi}\int_0^{2\pi} |f(e^{ix})|\,dx,$$

$$g(z) = 1+z^2, \quad z\in \boldsymbol{C},$$

$$h(z) = z, \quad z\in \boldsymbol{C},$$

$$L = \{cg:\ c\in \boldsymbol{C}\},$$

we find that for all $c\in \boldsymbol{C}$

$$\|h-cg\|_{H_1} = \frac{1}{2\pi}\int_0^{2\pi} |1-c\cdot 2\cos x|\,dx \geqslant \frac{1}{2\pi}\Big|\int_0^{2\pi} (1-c\cdot 2\cos x)\,dx\Big| = 1.$$

However, for $-1/2 \leqslant c \leqslant 1/2$ we have

$$1-c\cdot 2\cos x \geqslant 1-|c\cdot 2\cos x| \geqslant 1-|\cos x| \geqslant 0.$$

Thus,

$$\inf\{\|h-k\|_{H_1}:\ k\in L\} = \|h-cg\|_{H_1} = 1 \qquad \text{for } -1/2 \leqslant c \leqslant 1/2.$$

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