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Konstantin Mischaikow, Marian Mrozek and  
Piotr Zgliczynski (eds.)

**Conley Index Theory**

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## FOREWORD

The Conley index theory has undergone a lot of changes and development since it was constructed in the 60's and 70's. The monograph by Conley published in 1978, *On a generalization of the Morse index*, presented the state of the theory at that time, but since then there has been no global survey of further accomplishments. Therefore we gratefully accepted the offer from the Banach Center to organize a workshop on this subject.

The two-week workshop, which took place in Warsaw, Poland, in June 1997, was divided into two parts. The first week was concentrated on tutorial talks which gave the young participants an opportunity to get acquainted with the present state of the fundamentals of the theory. The second week was devoted to recent accomplishments, applications and connections to other theories.

In the first part of this volume the reader will find five tutorial papers: an introduction by K. Mischaikow, the foundations by R. Srzednicki, the index construction and properties by M. Mrozek, connection and transition matrices by Ch. McCord and J. Reineck, and the set-valued case by T. Kaczynski.

The rest of the volume contains research and survey papers presenting recent accomplishments in the theory and neighboring areas. The papers by Gidea, McCord, Mrozek–Srzednicki–Reineck and Pruszko present new developments in the index theory. The papers by Bartłomiejczyk–Dzedzej and Richeson concern the recent achievements in the connection matrix theory for discrete dynamical systems. There are three papers on the singular theory of the Conley index by Carbinatto, Hutson–Mischaikow and Kokubu–Mischaikow–Oka, and two papers on computational aspects of the theory by Osipenko and Zgliczyński. The paper by Kalies–Mischaikow–Watson concerns homology computation, a subject of vital importance in the numerical evaluation of the Conley index. Finally, there are two papers on border subjects: Morse–Floer homology by Rinaldi–Rybakowski and Nielsen numbers by Wójcik.

We hope that the volume will give the reader a perspective of the present state and directions of the Conley index theory.

*Konstantin Mischaikow*  
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*Piotr Zgliczyński*

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