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J. KUCHARCZYK (Wrocław)

ON THE CALCULATION OF RESERVE GENERATING CAPACITY IN ELECTRIC UTILITY SYSTEMS

0. Summary. The problem of determining reserve generating capacity in an electric utility system is presented from a rather new point of view. A simple model based on the cumulative distribution functions of customer demand and of power supply of each generating unit installed in the system is described. It is shown how to calculate the cumulative distribution function of power deficit, a function which contains all information necessary for calculating the reliability of power supply in the system. The idea of this model has been sketched earlier in [7].

1. Problem formulation. While planning the development of electric utility systems it is necessary to calculate how reliable future demand will be satisfied. This problem is known in the technical literature as the problem of reserve generating capacity. Generally speaking, it consists in determining that generating capacity of the system which will, with a given, fixed probability, assure the supply of power to customers (see [1]).

It is easy to see that it does not suffice for the nominal power capacities of all generators of the system to add to the greatest possible predicted demand; generators give usually less power into the system than their nominal capacity: Generators underlie breakdowns, their actual generating capacity may be diminished due to equipment failure (e.g. the outage of a steam boiler in a two-boiler-one-generator unit), and the need of generator maintenance calls for availability of additional power. Resuming, the system must have a power reserve to be able to satisfy even a known with certainty demand. In reality, however, the demand is known only approximatively, thus the problem becomes more involved. This paper presents a way of solution of the basic problem which consists in determining how much should the sum of all generating capacities of the units in the system exceed the greatest expected demand.

2. Known solution methods. Before presenting the model the solution methods hitherto published in the literature, which was available to the author, shall be briefly summarized. An excellent review of these methods may be found by the Polish reader in [16] (where also an introduction to the problem and a bibliography of nearly 100 items are given), good expositions of known methods and their application are also given in [8], [9], [10], [11], [14], [17].

Best known and most frequently applied are probability methods, usually referred to Calabrese ([4], [5]). Their main idea is to employ the binomial distribution to the calculation of the probability of outage of m from among n generators with a given, statistically determined, outage probability p. These probabilities are then used to find expected power outages which, together with both the expected customer demand and the capacities of generators being in repair, form the total demand for power in the system. This sum is compared with the capacity of all generators in the system and the reliability of power supply (or the probability of shortage of power) may be found.

Calculations with Bernoulli's formula are cumbersome, especially when large numbers of generators are involved. Therefore computational aids such as tables, desk calculators, or digital computers are needed, and their use reported (see e.g. [2], where some additional references may be found). On the other hand analytical simplifications have been proposed. They are of various nature, ranging from the use of fictitious systems (which behave like the real ones and are computationally easier to handle) instead of the real ones to the application of limit theorems (Poisson and normal distributions in place of binomial or multinomial ones).

Special methods, not based on the previously described idea, are also known. For instance, Seelye [12] and Watchorn [15], both have proposed methods using the idea of statistical regression. The knowledge of the values of some independent variables, such as the number of generators in the system, their generating capacities, outage probabilities, etc., enables the calculation of expected power capacity needs of the system in consideration.

Several types of power reserve are usually mentioned by different authors, their number varying from 3 to 6 and depending upon the number of factors considered (see e.g. [3]). Many authors agree that a complex method for the determination of the reliability of power supply is badly needed. Existing methods, each being appropriate for the calculation of another component of power reserve, do not allow a complex treatment of the problem. Now, a model will be presented which in the author's opinion permits a more complex consideration of the problem of reserve power capacity.

3. Presentation of the model. The model assumes that the distributions of available power capacity for each generator in the system and of customers power demand are known and independent of each other. The main simplification in the model is the assumption of what the technicians call current collector rail. Its meaning is that all power in the system is supplied from the generators directly to it and taken by the customers from it without any distribution lines. The purpose of this is to be able to neglect the role of the connection and distribution net both from the capacity as well as from the reliability viewpoints.

Denote by $Z(t)$ the power demand of all customers at moment t. Let the system consists of N generators, and let generator no. i have at moment t the power capacity given by $M_i(t)$. Knowledge of all of these functions would solve the problem of power reserve. It would be sufficient to calculate the power capacity of the whole system at moment t as the sum

$$M(t) = \sum_{i=1}^{N} M_i(t)$$

and then to find the difference between the demand and the supply at that moment:

$$Q(t) = Z(t) - M(t).$$

As long as $Q(t) \leqslant 0$, all customer needs are satisfied; power capacity shortage takes place when $Q(t) > 0$.

In practice the functions $M_i(t)$ and $Z(t)$ are not known. Both the available and the demanded power capacities of the system are random variables. Knowledge of the cumulative distribution functions of those random variables is assumed. Denote by $D(x)$ the cumulative distribution function of $Z(t)$; $D(x)$ is the relative measure of those t for which $Z(t) < x$, while t belongs to a known and fixed interval. Analogically, let $S_i(x)$ denote the cumulative distribution function of $M_i(t)$, i.e. the relative measure of those moments t for which $M_i(t) < x$.

On the basis of the functions $S_i(x)$ and $D(x)$ the function $G(x)$, the cumulative distribution function of power deficit $Q(t)$, will be calculated. Given $S(x)$, the cumulative distribution function of $M(t)$, $G(x)$, would be calculated as follows

$$G(x) = \int_{-\infty}^{\infty} S(x+y)\,dD(y).$$

The function $S(x)$ is a composition of all $S_i(x)$ $(i = 1, 2, \ldots, N)$:

$$S = S_1 * S_2 * \ldots * S_N.$$

This may be calculated successively, as follows. First, calculate

$$S_{1,2}(x) = S_1 * S_2 = \int_{-\infty}^{\infty} S_1(x-y)\,dS_2(y),$$

then add M_3 to the already regarded sum M_1+M_2:

$$S_{1,3}(x) = S_1 * S_2 * S_3 = \int_{-\infty}^{\infty} S_{1,2}(x-y)\,dS_3(y),$$

and so forth, till one obtains the distribution of the sum of all components of the system

$$S(x) = S_{1,N}(x) = S_1 * S_2 * \ldots * S_N = \int_{-\infty}^{\infty} S_{1,N-1}(x-y)\,dS_N(y).$$

Having $G(x)$ one is able to say whether the system in consideration is reliable at the fixed level, whether considerable power shortage is to be expected, or whether there will be unnecessary surplus of generating capacity in the system.

4. Information contained in $G(x)$. The cumulative distribution function of expected power deficit $G(x)$ contains, as already mentioned, all information needed for various performance characteristics and reliability indices of the system. Some of them are indicated in the following lines.

The quantity $1-G(0)$ determines the probability of any power shortage in the system, i.e., given any fixed time period for which calculations are made, this quantity shows the fraction of time with power shortage expected during this period. This is a traditionally used reliability measure of power supply in electric utility systems. If a shortage risk of duration not greater than 5 hours in a month is wanted, the system should assure that

$$1-G(0) \leqslant \frac{5 \text{ hours}}{1 \text{ month}} = \frac{5 \text{ hours}}{720 \text{ hours}} = 0.007,$$

thus

$$G(0) \geqslant 1-0.007 = 0.993.$$

If the considered system does not satisfy this condition, power capacity should be added to the system to assure the demanded reliability of power supply.

From the whole function $G(x)$ much more information may be retrieved. One is able to say what is the probability that there will be power shortage exceeding any given bound c, simply by calculating $1-G(c)$. This is of great practical value since small power shortages may be compensated through decrease of voltage or reduction of frequency (if allowed). On the other hand even considerable power capacity may be disconnected in certain systems for short time periods without harm to the customers involved. Suppose that the considered system has customers which allow the temporary disconnection of 10 megawatts and that 2 additional megawatts may be compensated by lowering the voltage; then a really dangerous situation arises only if the power deficit is greater than 12 megawatts. Thus the risk of shortage is measured by $1-G(12)$.

$G(x)$ permits also a calculation of the expected power deficit

$$d = \int_0^\infty x dG(x)$$

and the expected power surplus of the system

$$s = -\int_{-\infty}^0 x dG(x)$$

during the whole time period in consideration. Thus, having in mind that not only power shortage but also waste of power capacity cost money, the quantities d and s may serve as measures of the economics of a system. One may use here also other indices, as for instance

$$e = \int_{-\infty}^\infty |x| dG(x) = d+s.$$

H. Steinhaus [13] has proposed still another index of system reliability. It has the advantage of being independent of the size of the system and is defined as this ε for which holds

$$\Pr\{Q/E > \varepsilon\} = \varepsilon,$$

where

$$E = \int_{-\infty}^\infty x dD(x)$$

is the mean power demand. The parameter ε may easily be calculated from the equation

$$G(E\varepsilon) = 1-\varepsilon.$$

Being independent of the size of the utility system, the parameter ε may be used for comparisons of reliability levels of different systems or of the same system in its dynamic growth in time.

5. Computational remarks. The procedure of treating the reserve power capacity problem is easy to programme on any digital computer. Experiences concerning that question shall be published elsewhere. The model has been investigated also from the point of view of paper and pencil calculations, and a scheme for calculations with a desk calculator may be found in [6]. In this case the calculations are cumbersome but not more than those necessary in hitherto known methods. Limit theorems may be used as well to diminish the amount of work needed.

6. Additional remarks. The method presented is a generalization of earlier known methods. If only outage probabilities for the generators are given, as in Calabrese's method, the method may be used also since the cumulative distribution functions of power supply are then step functions with 2 steps. When more information about the power supply of generators is known, these functions will have more steps. In the extreme case they may be even of continuous shape. The availability of a computer programme for calculations described in sections 3 and 4 gives thus the possibility of using the method presented regardless of the amount of information on power supply at disposal.

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DEPARTMENT OF STATISTICS
UNIVERSITY OF WROCŁAW

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