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Symmetric Functions act on Exterior Algebras

A journey along Alain Lascoux's footprints

The magical joyful way Alain Lascoux looked at the ring of symmetric functions enables to extract non trivial, deep and beautiful mathematics even from the most classical and elementary notions. One example is provided by the euclidean algorithm of division of two univariate polynomials that, in its simplest application, allows the construction of a universal decomposition algebra $B_{r,n}$ of the monomial T^n as the product of two monic polynomials, one of degree $r \in \{0, 1, \ldots, n\}$. Using the natural action of the symmetric functions on the exterior algebra of a free abelian group of countable rank, the structure of $B_{r,n}$ as a module over the Lie algebra of $n \times n$ matrices will be described. This is related with the representation of the Lie algebra of infinite square matrices on a polynomial ring of infinitely many indeterminates, due to Date, Jimbo, Kashiwara and Miwa. The latter is just one of the items of the rich catalogue of difficult subjects tamed by Lascoux through the combinatorics of symmetric functions.