

Mike Roth

Title: Roth's theorem for arbitrary varieties, and other Diophantine applications of local positivity

Abstract: If  $X$  is a variety of general type defined over a number field  $k$ , then the Bombieri-Lang conjecture predicts that the  $k$ -rational points of  $X$  are not Zariski dense. The conjecture is a prediction that a global condition on the canonical bundle (that it is "generically positive") implies a global condition about rational points. By the local-global philosophy in geometry we should look for local influence of positivity on the accumulation of rational points. To do that we need measures of both these local phenomena. Let  $L$  be an ample line bundle on  $X$ , and  $x \in X(\bar{k})$ . The central theme of the talk is the interrelations between  $\alpha_x(L)$ , an invariant measuring the accumulation of rational points around  $x$  as gauged by  $L$ , and the Seshadri constant  $\epsilon_x(L)$ , measuring the local positivity of  $L$  near  $x$ . In particular, the classic approximation theorem of K.F. Roth on  $\mathbf{P}^1$  generalizes as an inequality between  $\alpha_x$  and  $\epsilon_x$  valid for all projective varieties. The talk will also discuss generalizations of these results to approximation higher dimensional subvarieties. This is joint work with David McKinnon.