Stefan Banach International Mathematical Center

8 Sniadeckich Str., 00-956 Warsaw 10, Poland

Phone: +48(0)22 628 01 92 Fax: +48(0)22 622 39 97

E-mail: banach@impan.gov.pl WWW: http://www.impan.gov.pl/BC

Institute of Mathematics of the Polish Academy of Sciences
Institute of Applied Analysis of the University of Ulm

Conference

FORMAL AND ANALYTIC SOLUTIONS OF DIFFERENTIAL EQUATIONS

August 11 – 16, 2008

Mathematical Research and Conference Center, Będlewo

Scientific Committee

Werner Balser (Ulm Uniwersity), Boele Braaksma (Rijksuniversiteit te Groningen), Masatake Miyake (Nagoya University), Reinhard Schäfke (Université de Strasbourg)

Organizing Committee

Werner Balser (Ulm) – Chairman, Grzegorz Łysik (Warsaw)
Sławomir Michalik (Warsaw).

♢ ♢ ♢

Aliasghar Jodayree Akbarfam (Tabriz University, Iran)

Duality in indefinite problem with three tuning points

The presentation consists of the four parts.

In the first part is given the review of the author works [1,3] devoted to representation of infinite product form of solution of Sturm Liouville equation with two turning point of types IV, II and with three turning point of types IV, II, III.

In the second part of the presentation we derive the dual equation of equation for each cases. Let us remind that forms of dual equations in definite and indefinite cases are deferent.

In the third part of the presentation we discus the existence and uniqueness solution of dual equations. Parts of these results are given in [2].

In the forth part of the presentation we investigate the inverse problem by making use of dual equations. We propose to apply interval analysis to obtain the potential function.

References

[1] A. Jodayree Akbarfam, A. B. Mingarelli, *The infinite product representation of solutions on indefinite Sturm-Liouville problems with two turning points*, The Arabian Journal for Science and Engineering, volume 31, number 1A.(2006).

[2] H. Kheiri, A. Jodayree Akbarfam, *The uniqueness of the solution of dual equations for Sturm- Liouville equation with turning point of order* $4m+1$, To be submitted.

[3] A. Jodayree Akbarfam, *The infinite product representation of solutions on indefinite Sturm- Liouville problems with three turning points*, To be submitted.

Werner Balser (University of Ulm, Germany)

Formal power series solutions

In the talk, the question of (multi-)summability of formal power series solutions to ordinary and partial differential, as well as other functional equations will be investigated.

◊ ◊ ◊

Adalberto Bergamasco (Universidade de Sao Paulo, Brazil)

Global solvability for a class of vector fields

We study the existence of global solutions for a class of complex vector fields in dimension two.

◊ ◊ ◊

Boele Braaksma (University Groningen, Netherlands)

Resurgence of formal solutions of meromorphic ODEs

The notion of resurgent function has been introduced by Ecalle: a resurgent function is an analytic function which can be analytically continued outside a lattice of singular points on the Riemann surface of the logarithm. Ecalle stated that formal solutions of meromorphic ODEs have Borel transforms which are resurgent. Their singular behavior is related to Stokes phenomenon. We discuss a proof by means of some simple cases.

Galina Filipuk (Loughborough University, UK)

Movable algebraic singularities of second order nonlinear differential equations

In this talk I shall consider movable algebraic singularities of second order ordinary differential equations. This is a joint work with RG Halburd (UCL, UK). For certain class of equations we show that the only movable singularities that can be reached by analytic continuation along finite-length curves are of the algebraic type (represented near a point z_0 by a Laurent series in fractional powers of $z - z_0$.) This generalizes previous result by Shimomura and Smith.

♢ ♢ ♢

Todor Gramchev (University of Cagliari, Italy)

Solvability for singular partial differential equations in anisotropic spaces of analytic–Gevrey functions

We investigate the solvability in scales of anisotropic spaces of analytic–Gevrey functions of classes of systems of singular partial differential equations appearing in normal form theory and Hamiltonian dynamics.

♢ ♢ ♢

Yoshishige Haraoka (Kumamoto University, Japan)

Prolongation of Fuchsian differential equations and computability of the monodromy

For Fuchsian differential equations free of accessory parameters, we have integral representations of solutions, which enables us not only to compute the monodromy groups but also to prolong the equations to regular holonomic systems in several variables. For example, Pochhammer equation has integral representation of solutions

$$y(x) = \int_p^q (t-a_1)^{b_1}(t-a_2)^{b_2}\cdots(t-a_n)^{b_n}(t-x)^c\,dt,$$

which is nothing but a specialization of Lauricella's F_D of $n-1$ variables. Thus Pochhammer euqation can be prolonged to F_D.

This suggests an existence of some relation among prolongability of an equation and computability of the monodromy. We shall give some stimulating examples of non-rigid (i.e. containing accessory parameters) equations, and discuss how this viewpoint will be fruitful. Relation to deformation theory is also discussed.

Kunio Ichinobe (Kanagawa IT, Japan)

Characterization of irregularity for singular systems of ordinary differential equations in complex domain

(Joint work with M. Miyake)

The irregularity of an ordinary differential operator with irregular singular point $z = 0$ is defined by the maximal rate of exponential growth of solutions of the associated homogeneous equation.

In this lecture, we shall study a singular N-system $L \equiv z^{p+1} d/dz - A(z)$ $(p \geq 0)$ with holomorphic coefficient matrix $A(z)$ near $z = 0$ and characterize its irregularity by order of zero of the coefficient matrix $A(z)$ at $z = 0$ in the sense of Volevič which was established by J. Moser, Jurkat-Lutz and others for the characterization of regular singular point.

♢ ♢ ♢

Geertrui Immink (University of Groningen, Netherlands)

Accelero-summability of formal solutions of nonlinear, analytic difference equations

The talk is concerned with nonlinear, locally analytic difference equations. It has been known for some time that formal power series solutions of such equations are not necessarily multi-summable. I shall discuss a recent result to the effect that these formal solutions do admit a unique sum, which can be obtained by a process of accelero-summation.

♢ ♢ ♢

Jan Janas (Polish Academy of Sciences, Poland)

Asymptotic of generalized eigenvectors of unbounded Jacobi matrices

Some asymptotic results on generalized eigenvectors of unbounded Jacobi matrices and their applications in the spectral theory of these matrices will be presented.

EKATERINA KUTAFINA (University of Mining and Metallurgy, Cracow, Poland)

Exact solutions of hyperbolic generalization of Burgers equation

Hyperbolic generalization of Burgers equation occurs when the memory effects are taken into account. We consider the set of solutions of the form

$$u(x,t) = \frac{M(x,t)\,Exp(h(x,t)) + Q(x,t)}{Exp(h(x,t)) + 1}.$$

Inside this set we introduce certain structure of equivalence classes and as a result we obtain a possibility to join a pair of known solutions to build a new one. The non-trivial solutions are found.

♢ ♢ ♢

DONALD LUTZ (San Diego State University, USA)

Some perspectives on the development of meromorphic differential equations

This talk will concern various facets in the development of the theory of meromorphic differential equations. After a brief historical summary and description of the types of problems encountered, some of the main ideas which have contributed to the present state theory will be discussed. Finally, a personal assessment of how the present state compares to some of the original goals will be made.

♢ ♢ ♢

GRZEGORZ ŁYSIK (Polish Academy of Sciences, Poland)

Formal and analytic solutions of the Burgers equation

In the first part of the lecture we shall study formal power series solutions to the initial value problem for the Burgers type equation $\partial_t u - \Delta u = X(f(u))$ with polynomial nonlinearity f and a vector field X, and we prove that they belong to the formal Gevrey class G^2.

Next we give necessary and sufficient conditions for the formal power series solutions to the initial value problem of the Burgers equation $\partial_t u - \partial_x^2 u = \partial_x(u^2)$, be convergent or Borel summable.

Stephane Malek (University of Lille, France)

On complex singularity analysis of q-differential difference equations

We investigate the existence of global holomorphic solutions of linear q-difference-differential equations, with meromorphic coefficients, having complex singularities along q-spirals. We also study the rate of growth of the constructed solutions near these singular points.

♢ ♢ ♢

Sławomir Michalik (Cardinal Stefan Wyszyński University, Poland)

Summability of formal solutions of partial differential equations with constant coefficients

We shall consider the formal power series solutions to the Cauchy problem for non-Kowalevskian linear partial differential equations with constant coefficients. We shall give necessary and sufficient conditions for the Borel summability in terms of the Cauchy data.

♢ ♢ ♢

Gerard Misiołek (University of Notre Dame, USA)

A new integrable equation on the Virasoro group
(Joint work with Boris Khesin and Jonatan Lenells)

The equation is related to the KdV, CH and HS equations. I will discuss its derivation, bi-hamiltonian structure and soliton solutions. I will also present what is currently known about well-posedness of the corresponding Cauchy problem.

Masatake Miyake (Nagoya University, Japan)

Newton polygon for a singular system of ordinary differential equations

We shall study the following singular system of Poincaré rank $p \geq 1$ at $z = 0$,

$$L = z^{p+1} D_z - A(z), \quad A(z) = (a_{ij}(z))_{i,j=1,2,\cdots,N},$$

where $z \in \mathbb{C}$, $D_z = d/dz$ and $a_{ij}(z)$ are holomorphic functions in a neighborhood of $z = 0$.

The purpose of of my lecture is to define the Newton polygon of L.

Let $Y(z)$ be Hukuhara-Turrittin's formal fundamental matrix solutions, which is obtained after a long reduction procedures into a canonical form and is written in the form,

$$Y(z) = \hat{Y}(z) z^G \exp[Q(z)],$$

where $\hat{Y}(z)$ is a formal power series of fractional power of z, G is a constant matrix and $Q(z)$ is a diagonal matrix with entries of polynomials of a fractional power of $1/z$ which is called the determining factor. If we want to know the nature of singularities at $z = 0$, it will be obtained from $Q(z)$, but it is not so easy to obtain exactly (cf. the text book by W. Wasow, Theorem 19.1, pp. 111-112).

The Newton polygon of L will be defined to describe the leading terms of polynomials in $Q(z)$, which is done by reducing into a precanonical form of $L(z, D_z)$ by an invertible matrix of rational functions with possible poles at $z = 0$.

♢ ♢ ♢

Giovanni Morando (Università degli Studi di Padova, Italy)

Subanalytic site and tempered holomorphic solutions of $\mathcal{D}$-modules on a complex curve

In this talk we will recall the definition of the subanalytic site and of the subanalytic sheaf of tempered holomorphic functions. Then we will use these objects in the study of holonomic $\mathcal{D}$-modules on a complex curve.

In detail, given a linear ordinary differential equation, we will describe its classical invariants (given by the formal decomposition and the Stokes matrices) by means of its tempered solutions. We will underline the link with the real spectra used in the semi-algebraic case. Further we will prove an existence theorem for tempered holomorphic solutions which implies the $\mathbb{R}$-constructibility of the complex of tempered holomorphic solutions of a holonomic $\mathcal{D}$-module on a complex curve. This last result have been conjectured in all dimensions by M. Kashiwara and P. Schapira.

Jorge Mozo-Fernández (University Valladolid, Spain)

Computing liouvillian solutions of linear ODEs

(Joint work with Alberto Llorente)

Liouvillian solutions of linear ordinary differential equations over a differential field K correspond to straight lines invariant by G^0, the identity component of the differential Galois group G of the equation. The group G, according with Ramis' density theorem, can be generated by a explicit finite number of elements (formal monodromy, exponential torus and Stokes operators), in the case where $K = \mathbf{C}(X)$, in which we are interested. These generators can be computed in the framework of effective analysis developed by J. van der Hoeven.

In order to compute a liouvillian solution, we shall see how the group G can be extended to a bigger group $\bar{G}$, computable at ε precision, and that inherits some properties of G. In particular, either a liouvillian solution can be computed if it exists, or it can be shown that such a solution does not exist. The main problems come from the membership or not of an element to G^0, and from rational resonances.

$\diamondsuit \quad \diamondsuit \quad \diamondsuit$

Luca Prelli (Università degli Studi di Padova, Italy)

Sheaves on subanalytic sites and D-modules

Sheaf theory is not well suited to study objects which are not defined by local properties. It is the case, for example, of functional spaces with growth conditions, as tempered distributions. In 2001 Kashiwara and Schapira introduced the subanalytic site and proved that some of this spaces can be realized as sheaves on a subanalytic site. In this talk we will recall the theory of subanalytic sheaves and give an overview of some recent developments. In particular we construct functorially the sheaf of function asymptotically developable using holomorphic functions with growth conditions.

Claudia Röscheisen (Ulm University, Germany)

On hypergeometric systems of differential equations

In this talk, we will introduce two sequences of functions and give some properties of these functions. Furthermore, we will show how solutions and characteristic constants of the hypergeometric system of differential equations can be expressed in terms of these functions.

♢ ♢ ♢

Javier Sanz (Universidad de Valladolid, Spain)

Summability in strong asymptotics

We will describe a notion of **k**-summability in a direction for formal power series in several variables, departing from the concept of strong asymptotics introduced by H. Majima and particularized by Y. Haraoka to the Gevrey case. After stating the equivalence of this notion to an iterative summability method (that will be described), we apply it to a singular perturbation problem already discussed by J. Ecalle and W. Balser. Finally we will show how the sum of a formal power series can be given by means of a multidimensional Laplace transform.

♢ ♢ ♢

Reinhard Schäfke (Strasbourg University, France)

TBA

♢ ♢ ♢

Dieter Schmidt (University of Duisburg-Essen, Germany)

On the global solutions of the Confluent HEUN Equation (CHE)
(Joint work with Stephan Schultze)

The Confluent HEUN Equation (CHE) is the linear second-order differential equation with three singular points, namely two simple singularities at 0 and 1 and an irregular singularity of rank one at infinity. In the talk we shall investigate the Frobenius and the Floquet solutions of the CHE at the simple singularities 0 and 1 in form of series in terms of Hypergeometric functions and further the asymptotic solutions at infinity in form of series in terms of Confluent Hypergeometric functions. We discuss connection relations between these solutions as well as between the expansion coefficients of the different series representations.

Akira Shirai (Sugiyama Jogakuen University, Japan)

Alternative proof for the convergence of formal solutions of singular first order nonlinear partial differential equations

Let $(t, x) \in \mathbb{C}^d \times \mathbb{C}^n$. We consider the following first order nonlinear partial differential equation: $f(t, x, u, \partial_t u, \partial_x u) = 0$, $u(0, x) \equiv 0$ where $u = u(t, x)$ is unknown function, $\partial_t u = (\partial_{t_1} u, \ldots, \partial_{t_d} u)$ and $\partial_x u$ is similar with $\partial_t u$. In 2005, M. Miyake and I studied the convergence of formal solution of above equation under some conditions from the viewpoint as evolution equation in t variables.

The purpose of this talk is to give an alternative proof for the convergence of formal solutions. The proof in this talk is from the viewpoint that the role of variables t and x are equivalent, which is often called the totally characteristic type.

$\diamondsuit \quad \diamondsuit \quad \diamondsuit$

Hidetoshi Tahara (Sophia University, Japan)

Coupling of two singular partial differential equations and its application

In this talk I will consider a reduction of a singular Briot-Bouquet type partial differential equation (A) $t\partial u/\partial t = F(t, x, u, \partial u/\partial x)$ to a simple form (B) $t\partial w/\partial t = \lambda(x)w$ with $\lambda(x) = (\partial F/\partial u)(0, x, 0, 0)$ in the complex domain under the assumption that (A) satisfies certain Poincaré condition. The reduction is done by considering the coupling of two equations (A) and (B), and by solving their coupling equation. The result is applied to the problem of finding all the singular solutions of (A).

Feride Tiglay (University of New Orlean, USA)

Existence of some infinite energy solutions to Euler equations
(Joint work with R. Saxton)

A class of solutions to the Euler equations for a perfect, incompressible fluid consists of those having "stagnation-point" form, which attracted early attention by Weyl and Lin and provides a set of equations which depend on only a single spatial variable and time. The resulting equations, once solved, provide exact solutions to the full Euler equations. However, the associated growth of the full solutions in certain directions means that the flows possess, at best, only locally finite kinetic energy. In this talk we will discuss a recent result on local and global existence for such a class of solutions to the Euler equations for an incompressible, inviscid fluid.

♢ ♢ ♢

Hideshi Yamane (Kwansei Gakuin University, Japan)

Logarithmic singularities of solutions to nonlinear partial differential equations
(Joint work with Hidetoshi Tahara)

We construct a family of singular solutions to some nonlinear partial differential equations. The leading term of a solution in our family contains a logarithm, possibly multiplied by a monomial.

The proof is done by the reduction to a Fuchsian equation with singular coefficients.

♢ ♢ ♢

Masafumi Yoshino (Hiroshima University, Japan)

C^ω Liouville-nonintegrable and C^∞ Liouville-integrable Hamiltonian systems

We will show that there exists a class of C^ω-Liouville-nonintegrable and C^∞-Liouville-integrable Hamiltonian systems. We show that the monodromy property of a certain susbsystem of the given Hamiltonian system is closely related with the nonintegrability. We also show that the hyperasymptotic expansions and the summability argument play an important role in the study of integrability. (A part of the talk is published in Annali di Matematica Pura ed Applicata.)

List of participants and their e-mail addresses

AKBARFAM ALIASHAR JODAYREE (Tabriz University): akbarfam@yahoo.com
ANDO KANA (Université d'Angers): ando@math.univ-angers.fr
BALSER WERNER (Ulm University): werner.balser@uni-ulm.de
BERGAMASCO ADALBERTO (Universidade de Sao Paulo): apbergam@icmc.usp.br
BRAAKSMA BOELE (University of Groningen): braaksma@math.rug.nl
BOJARSKI BOGDAN (Polish Academy of Sciences) bojarski@impan.gov.pl
DUVAL ANNA (USTL Lille): duval@math.univ-lille1.fr
TODOR GRAMCHEV (University of Cagliari): todor@unica.it
HARAOKA YOSHISHIHE (Kumamoto University): haraoka@kumamoto-u.ac.jp
ICHINOBE KUNIO (Kanagawa Institute of Technology): ichinobe@gen.kanagawa-it.ac.jp
IMMINK GEERTRUI (University of Groningen): g.k.immink@rug.nl
JANAS JAN (Polish Academy of Sciences): najanas@cyf-kr.edu.pl
KUTAFINA EKATERINA (University of Mining and Metallurgy): kutafina@matagh.edu.pl
LASTRA ALBERTO (Universidad de Valladolid): alastra@am.uva.es
LUTZ DONALD (San Diego State University): lutzmath@hotmail.com
GRZEGORZ ŁYSIK (Polish Academy of Sciences): lysik@impan.gov.pl
MALEK STEPHANE (University of Lille): Stephane.Malek@math.univ-lille1.fr
SŁAWOMIR MICHALIK (Cardinal Stefan Wyszyński University, Warsaw):
s.michalik@uksw.edu.pl
MISIOLEK GERARD (University of Notre Dame): gmisiole@nd.edu
MIYAKE MASATAKE (Nagoya University): mmiyake@math.nagoya-u.ac.jp,
kfhnt984@ybb.ne.jp
MORANDO GIOVANNI (Universitá di Padova): gmorando@math.unipd.it
MOZO-FERNANDEZ JORGE (Universidad de Valladolid): jmozo@maf.uva.es
PRELLI LUCA (University of Padova): lprelli@math.unipd.it
RÖSCHEISEN CLAUDIA (University of Ulm): claudia.roescheisen@uni-ulm.de
SANZ JAVIER (Universidad de Valladolid): jsanzg@am.uva.es
SCHMIDT DIETER (University of Duisburg-Essen): dieter.schmidt@uni-DuE.de
SCHÄFKE REINHARD (Strasbourg University): schaefke@math.u-strasbg.fr
SHIRAI AKIRA (Sugiyama Jogakuen University): shirai@sugiyama-u.ac.jp
STENGER CATHERINE (University of La Rochelle): cstenger@univ-lr.fr
TAHARA HIDETOSHI (Sophia University): h-tahara@sophia.ac.jp
TIGLAY FERIDE (University of New Orleans): ftiglay@uno.edu
YAMANE HIDESHI (Kwansei Gakuin University): yamane@kwansei.ac.jp
YOSHINO MASAFUNI (Hiroshima University): yoshinom@hiroshima-u.ac.jp