The variational principle for a continuous map T on a compact space Y, with respect to some Hölder potential ϕ, asserts that the pressure of ϕ coincides with the supremum over all invariant probability measures μ of the free energy $h_\mu(T) + \int \phi\, d\mu$. There is a relative version of the variational principle due to F. Ledrappier and P. Walters. Given a transformation $S\colon X \to X$ over which T fibers, and an invariant measure ν on X, this principle expresses the supremum of the relative free energy over invariant probability measures projecting on ν as the integral of the pressure on the fibers with respect to ν.

The transformation T is said to be fiber expanding if the restrictions $T_x\colon \pi^{-1}(x) \to \pi^{-1}(Sx)$ are expanding with respect to some metric d on Y: $d(u,v) < \lambda d(Tu, Tv)$ for u, v on the same fiber. It is exact on fibers if $\pi^{-1}(S^n(\pi(y)) \subset T^n(B(y,\varepsilon) \cap \pi^{-1}(\pi y))$, for all $y \in Y$ and n large. Suppose, moreover, that T is bounded-to-one on fibers.

Under these assumptions, we give a new expression of the supremum of the free energy in terms of a gauge function defined using relative transfer operators. We show that this supremum is finite and attained for a unique T-invariant probability measure.