On the Hausdorff dimension of the escaping set of certain meromorphic functions

Let f be a transcendental meromorphic function of finite order ρ for which the set of finite singularities of f^{-1} is bounded. Suppose that ∞ is not an asymptotic value and that there exists $M \in \mathbb{N}$ such that the multiplicity of all poles, except possibly finitely many, is at most M. For $R > 0$ let $I_R(f)$ be the set of all $z \in \mathbb{C}$ for which $\liminf_{n\to\infty} |f^n(z)| \geq R$ as $n \to \infty$. Here f^n denotes the n-th iterate of f. Let $I(f)$ be the set of all $z \in \mathbb{C}$ such that $|f^n(z)| \to \infty$ as $n \to \infty$; that is, $I(f) = \bigcap_{R>0} I_R(f)$. Denote the Hausdorff dimension of a set $A \subset \mathbb{C}$ by $\text{HD}(A)$. It is shown that $\lim_{R\to\infty} \text{HD}(I_R(f)) \leq 2M\rho/(2 + M\rho)$. In particular, $\text{HD}(I(f)) \leq 2M\rho/(2 + M\rho)$. These estimates are best possible: for given ρ and M we construct a function f such that $\text{HD}(I(f)) = 2M\rho/(2+M\rho)$ and $\text{HD}(I_R(f)) > 2M\rho/(2 + M\rho)$ for all $R > 0$.

If f is as above but of infinite order, then the area of $I_R(f)$ is zero. This result does not hold without a restriction on the multiplicity of the poles