Speaker: Michal Rams (IMPAN)

Title: Inhomogeneous Diophantine approximation with general error functions.

Abstract: The problem: given an irrational number α and a nonincreasing sequence $\phi(n)$, how big (in terms of Hausdorff dimension) is the set $E_\phi(\alpha) := \{y \in S^1; ||n\alpha - y|| < \phi(n) i.o.\}$.

It was partially solved by Bernik & Dodson and then completely by Bugeaud and by Schmeling & Troubetzkoy for $\phi(n) = n^{-\gamma}$. In general situation, $\liminf(\log n / -\log \phi(n)) \leq \dim_H E_\phi(\alpha) \leq \limsup(\log n / -\log \phi(n))$ for all α. Last year, Xu proved that for α of Diophantine type 1 $\dim_H E_\phi(\alpha) = \limsup(\log n / -\log \phi(n))$, but Fan and Wu constructed a number α and sequence $\phi(n)$ for which $\dim_H E_\phi(\alpha) = \liminf(\log n / -\log \phi(n)) < \limsup(\log n / -\log \phi(n))$.

I'm going to present my joint work with Lingmin Liao, in which we give the best possible estimations on $\dim_H E_\phi(\alpha)$ given $\liminf(\log n / -\log \phi(n))$, $\limsup(\log n / -\log \phi(n))$ and the Diophantine type of α.