

Abstract: I'll present my work with Jairo Bochi on the products of  $GL(2, R)$  matrices. Setting: given a finite set of  $GL(2, R)$  matrices  $\{A_i\}_{i=1}^k$ , we consider for all possible sequences  $\omega \in \{1, \dots, k\}^{\mathbb{N}}$  the maximal Lyapunov exponent

$$\lambda(\omega) = \lim_{n \rightarrow \infty} (1/n) \log \|A^{(n)}(\omega)\|,$$

where  $A^{(n)}(\omega) = A_{\omega_n} \cdot \dots \cdot A_{\omega_1}$ . We investigate the interval  $[\lambda_-, \lambda_+]$  of possible values of  $\lambda$ . We prove that under specific assumptions (domination + nonoverlapping) the extremal values of  $\lambda$  are attained, thus there exist invariant measures on  $\{1, \dots, k\}^{\mathbb{Z}}$  with Lyapunov exponents  $\lambda_-$  or  $\lambda_+$ , but all those measures have zero entropy. Contrary to that, at least in the  $SL(2, R)$  case, without domination it is an open and dense property for the invariant measures with maximal Lyapunov exponent equal to  $\lambda_-$  to have positive entropy.