Speaker: Julia Romanowska (The University of Warsaw)
Title: On the dimension of the graph of the classical Weierstrass function

Abstract: In my talk I will examine dimension of the graph of the famous Weierstrass non-differentiable function

$$W_{\lambda,b}(x) = \sum_{n=0}^{\infty} \lambda^n \cos(2\pi b^n x)$$

for an integer $b \geq 2$ and $1/b < \lambda < 1$. In our recent paper, together with Balázs Bárány and Krzysztof Barański, we prove that for every b there exists (explicitly given) $\lambda_b \in (1/b, 1)$ such that the Hausdorff dimension of the graph of $W_{\lambda,b}$ is equal to $D = 2 + \frac{\log \lambda}{\log b}$ for every $\lambda \in (\lambda_b, 1)$. We also show that the dimension is equal to D for almost every λ on some larger interval. This partially solves a well-known thirty-year-old conjecture. Furthermore, we prove that the Hausdorff dimension of the graph of the function

$$f(x) = \sum_{n=0}^{\infty} \lambda^n \phi(b^n x)$$

for an integer $b \geq 2$ and $1/b < \lambda < 1$ is equal to D for a typical Z-periodic C^3 function ϕ.