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## Institute of Mathematics of the Polish Academy of Sciences
## Institute of Mathematics of the University of Rzeszów

## Conference

# LINEAR AND NON-LINEAR THEORY OF GENERALIZED FUNCTIONS AND ITS APPLICATIONS

September 2 – 8, 2007

**Mathematical Research and Conference Center, Będlewo**

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**The Banach Center Conference**

# Linear and Non-linear Theory of Generalized Functions and Its Applications.

**Mathematical Research and Conference Center, Będlewo**

September 2 - 8, 2007

## ABSTRACTS

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Jelena Aleksić (University of Novi Sad)

**Equivalence between Colombeau's approximate solutions and measure valued solutions to conservation laws**

The purpose of this talk is to relate the concept of measure valued solutions to conservation laws, introduced by Di Perna, to the concept of approximate and generalized solutions, arising in differential algebras, containing the distributions and having the algebra of smooth functions as a subalgebra, called Colombeau's algebras.

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Anatolij Antonevich (Belorusian St. Univ.)

**On the equations with a delta-shaped coefficients**

Differential expressions of the form

$$Lu = L_0 u + a\delta u, \tag{1}$$

where $L_0$ is a partial differential operator and $\delta$ is the Dirac distribution, emerge from the mathematical modelling of the scattering on a particle, located at the one fixed point. The expression (1) is formal and the first problem is to construct corresponding operator. Giving the sense to the expression (1) as an operator in the space $L_2(\mathbb{R}^3)$ (which is necessary in quantum theory) requires to overcome some obstacles. This related with the fact, that the product $\delta \cdot u$ is not defined in the classical distribution's theory and some ideas from the theory of new generalized function must be used.

In the lecture proposed three approaches will be presented to solve this problem.

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Piotr Antosik (Polish Academy of Sciences)
**On Mikusiński's seminar in Katowice**
[joint presentation with Andrzej Kamiński]

The topics of the seminar were closely connected with mathematical interests of Professor Jan Mikusiński and included the operational calculus, generalized functions, convergence structures and integration theory. In 1966 the members of the seminar organized in Katowice an international conference on generalized functions, in which about seventy mathematicians took part, among them so eminent as S. Sobolev, L. Schwartz, J. Dieudonne, G. Temple, H. Komatsu, T. Boehme and others.

Members of the seminar were invited to many mathematical centers all over the world. In the period of the duration of the seminar (1960-1987) its members published several books (more than 20 editions altogether) and about 250 scientific papers. Eight members of the seminar received their Ph.D.'s, two of them were habilitated and then two were promoted to the position of full professor. Spontaneous contacts of the seminar with the mathematical community in the world gave impetus for establishing the Katowice Branch of the Institute of Mathematics of the Polish Academy of Sciences.

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Nadzeya Bedziuk, Aleh Yablonski (Belorussian St. Univ.)
**The equations in differentials in algebra of generalized functions**

Solutions of differential equations with generalized coefficients essentially depend on their interpretation. In this work we show that such different interpretations can be described by using equation in differentials in algebra of new generalized functions.

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Andrzej Borys (University of Technology and Life Sciences)
**Volterra series and multiplication of Dirac impulses**
[joint work with Wiesław Sieńko]

Volterra series and Volterra-Wiener system representations are basic tools for input-output description of nonlinear systems. Roughly speaking, Volterra systems are described by infinite sums of homogeneous terms (systems) of degree $n > 1$ . For such systems impulse inputs (expressed by the Dirac delta distribution) cause mathematical problems when a direct transmission term is presented, because they lead to undefined objects in the form of the $n-$th power of the Dirac delta distribution in the response. It is well known that direct transmission terms arise from unintegrated input signals in the nonlinear part of systems. Our aim is to present some aspects of the physical realizability of nonlinear systems, for example, an identification of kernels of degree $n$ using impulse inputs.

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Chikh Bouzar (Oran-Essenia University)

**Ultra-regular generalized functions**

Algebras of ultradifferentiable generalized functions satisfying some regularity assumptions are introduced. We give a micro-local analysis within these algebras related to the regularity type and the ultradifferentiability property. As a particular case we obtain new algebras of regular Gevrey generalized functions.

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Ekaterina Bozhonok (Taurida National University)

**Pseudoquadratic variation functionals in Sobolev space $W_2^1$**

It is known that in $W_2^1 := W_2^1([a;b], E)$, where $E$ is a Banach space, the basic variation functional

$$\Phi(y) = \int_a^b f(x, y, y')\, dx, \qquad y \in W_2^1$$

has special differential and extremal properties.

A thorough analysis of a situation shows that well-definiteness conditions of the basic variation functional in Sobolev space are already connected to the additional requirement of "pseudoquadraticity" of integrand with respect to $y'$.

We give sufficient conditions of well posedness, $K-$continuity, $K-$differentiability and twice $K-$differentiability for Euler-Lagrange functional in $W_2^1$ in terms of "pseudoquadratic functionals". An example of $K-$continuous integral functional is considered.

**Reference**

[1] I. V. Orlov, E. V. Bozhonok, *Well–definiteness, $K$–continuity and $K$–differentiability conditions for Euler–Lagrange functional in*

*Sobolev space* $W_2^1$, Scientific Notes of Taurida National University, **19** (58) (2006), 121-136 (*in Russian*).

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Aneta Bučkovska (Ss Cyril and Methodius University, Skopje)
**Inversion theorem for bilinear Hilbert transform**
[joint work with Stevan Pilipović (Novi Sad)]

An approximation result for the bilinear Hilbert transform is proved and used for the inversion of the bilinear Hilbert transform. Also, $p$-Lebesgue points ($p \geq 1$) are analyzed.

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Józef Burzyk (Silesia Technical University)
**To be announced**

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Soon-Yeong Chung (Sogang University)
**Laplace equations on nonlinear networks and their inverse problems**

We introduce $p$-Laplace operators and $p$-harmonic functions defined on nonlinear networks and discuss the solvabilities of the Dirichlet boundary value problems and Neumann boundary value problems. This talk will be focused on the uniqueness of the inverse problem which is to identify the conductivity of links between adjacent pairs of nodes in a nonlinear network with the help of data measured on the boundary of the networks. Besides, several interesting feature of partial differential equations on networks will be discussed.

**References**

[1] H. Bellout and A. Friedman, *Identification problems in potential theory*, Arch. Rational Mech. Anal. **101** (1988), 143-160.

[2] N. L. Biggs, *Algebraic graph theory on graphs*, Bull. London Math. Soc. **29** (1997), 641-682.

[3] F. R. K. Chung, *Spectral graph theory*, CBMS Regional Conference Series in Math. **92**, Amer. Math. Soc. 1997.

[4] S.-Y. Chung, and C. Berenstein, *$\omega$-Harmonic functions and inverse conductivity problems on networks*, SIAM J. Appl. Math. **65** (2005), 1200-1226.

[5] S.-Y. Chung, J.-H. Kim, and Y.-S Chung, *Diffusion and elastic equations on networks*, to appear in Publ. RIMS, 2007.

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Yun-Sung Chung (Sungkyunkwan University)

**An inverse conductivity problem for evolution equations on networks**

In this talk, we discuss an inverse problem of identifying the connectivity and the conductivity of the links between adjacent pair of nodes in networks satisfying a certain type of equations, called the evolution equations on networks by using their boundary measurements. Since the inverse problem assumes a direct problem, we first deal with the existence and the uniqueness of (direct) problems such as the Cauchy problems and the Dirichlet boundary value problems for the evolution equations on networks. The main result is the global uniqueness of the inverse conductivity problems under a monotonicity condition.

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Jean-François Colombeau (Wanadoo)

**Nonlinear generalized functions and the Heisenberg-Pauli calculations**

[to be delivered during the Conference]

The Heisenberg-Pauli calculations (1929) are a set of three or four pages of calculations (in a simple yet fully representative model) that are formally quite easy and mimic calculations on functions. They are a basic formulation in Quantum Field Theory: "canonical Hamiltonian formalism" [Weinberg, The Quantum Theory of Fields, vol. l, Cambridge, 1995, p. 292: all known physically significant QFT are obtained by treating a lagrangian density with the H-P calculations]. They start with irregular distributions that are multiplied, exponentiated, etc. Since 1955 it has been taught to physicists that mathematicians have "proved" (i.e. with a rigorous mathematical proof!!!) that "there will never exist a nonlinear theory of generalized functions, in any mathematical context possibly different from the theory of distributions" [Schwartz, Thorie des distributions, Paris, 1966, p. 10]. In the last 50 years mathematical physicists have unsuccessfully attempted to replace the H-P calculations by something that could make sense within the distributions, while theoretical physicists went on using the H-P calculations in various theories from "Quantum Electrodynamics (1930-1947)" to the present "Standard Model of particle physics (about 1990)". In the first part of the talk we discuss the above "proof". In the second part we explain the H-P calculations. They make sense mathematically in the nonlinear theory of generalized functions. But only a too coarse work has been done. The H-P calculations request a deeper study: they concern generalized functions whose values are unbounded operators on a Hilbert space, which makes them more intricate than scalar valued calculations, as those done in continuum mechanics [Colombeau, arXiv math-ph/0702014, see also arXiv math-ph/0612077 and arXiv math-FA/0610264] and general relativity [Steinbauer-Vickers, Class. Quant. Grav. **23** (2006), 91-l14 or arXiv.org gr-qc/0603078]. Numerous presumably easy problems of pure mathematics and other ones presumably difficult are stated. Further, a listener interested in computer calculations can attempt the calculation of the new numerical predictions issued from the mathematical understanding of the H-P formalism: it shows how to perform "nonperturbative" calculations. A copy of the detailed (formal, and then rigorous) calculations will be put at the disposal of any interested listener; absolutely no knowledge of physics is needed; it suffices to know the definitions of generalized functions in the "simplified" or "special" case and to be interested in unbounded operators on the Hilbert space. These problems should be solved before a presentation to theoretical physicists (i.e. mathematicians in pure mathematics and other ones in computer calculations should clarify simple mathematical models before one could propose theoretical physicists to reproduce their work on physically significant theories not accessible to mathematicians). Even if a future String Theory replaces Quantum Field Theory a mathematical understanding of QFT would be needed [Witten, Bull. AMS **40** (2002), 21-29]. So the proposed works and problems are presumably very important for the future. They are suited for mathematicians unaware of physics.

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JEAN-FRANÇOIS COLOMBEAU (Wanadoo)
**Generalized functions as a tool for nonsmooth nonlinear problems in mathematics and physics**
[the subject of this abstract may be discussed with the author]

A differential algebra of nonlinear generalized functions is presented as a tool for a wide range of nonsmooth nonlinear problems. The power of the differential algebra is used to do mathematical calculations or proofs; then the final result is often a classical function or distribution which is not solution in the classical or distributional sense. The aim of the lecture is to invite listeners in applying this tool in their own research without significant prerequisites by presenting its use on a sample of elementary applications from mathematics and physics.

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ANETA DADEJ & KATARZYNA HALIK (University of Rzeszów)
**Product of distributions via Hermite expansions**

P. Catuogno, S. Molina and C. Olivera introduced in the note *On Hermite representation of distributions and products*, Integral Transforms and Special Functions **18** (2007), 233-243 the product of tempered distributions using their Hermite expansions. We call this type of the product the *Hermite product* of distributions. In our presentation we consider also other versions of the Hermite product of tempered distributions and compare all of them with some of the known definitions of the product of distributions.

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VLADIMIR DANILOV (Moscow Technical University)
**On the relation between the Maslov–Whitham method and the weak asymptotics method**

Let $\omega \in \mathcal{S}(\mathbb{R}^1)$, $\beta, \varphi$ be $C^1$-functions, and $\psi \in \mathcal{S}(\mathbb{R}^1)$ be a test function. We consider the expression

$$\begin{aligned}\langle \omega\Big(\beta\frac{x-\varphi}{\varepsilon}\Big), \psi\rangle &= \int \omega\Big(\beta\frac{x-\varphi}{\varepsilon}\Big)\psi(x)\,dx \\ &= \sum_{k\geq 0} \frac{\varepsilon^{k+1}}{\beta^{k+1}k!}\Omega_k(-1)^k\langle\delta^{(k)}(x-\varphi), \psi\rangle,\end{aligned}$$

where $\Omega_k := \int_{\mathbb{R}^1} z^k\omega(z)\,dz$.

The sum in the right-hand side is understood as an asymptotic series. The first relation appears, for example, in constructing weak asymptotic solutions of soliton type, see [1], the second is the well-known momentum decomposition in the theory of algebras of Colombeau generalized function.

In the framework of the weak asymptotics method, we studied many problems of interaction of solitary nonlinear soliton- and kink-type waves in equations with a small parameter at the highest-order derivatives. In these cases, the smoothing of the corresponding limit generalized function ($\varepsilon\delta(x-\varphi)$ for solitons and $H(x-\varphi)$ for kinks) remained beyond the framework of the weak asymptotics method. At the same time, it is known that these smoothings ($\omega(z)$, see above) are determined as solutions of ODE called standard. For example, in the case of the KdV equation, we know $\omega(z) = \cosh^{-2}(z)$. The problem is that in the weak asymptotics method, we take the first (leading) term in series of the form (1), while the standard equation is obtained if we calculate all the terms of the series and additionally assume that $\omega(z)$ is an analytic function (then $\omega(z)$ is determined by the momenta $\Omega_k$).

In order to find the relation between the Maslov–Whitham method and the weak asymptotics method, we note that

$$\langle\omega\Big(\beta\frac{x-\varphi}{\varepsilon}\Big),\psi\rangle = \Omega\Big(-\frac{i\varepsilon}{\beta}\frac{\partial}{\partial x}\Big)\psi\Big|_{x=\varphi}, \tag{1}$$

where $\Omega(p) = F_{z\to p}\omega(z)$ is the Fourier transform.

It is easy to see that, in the form of the right-hand side of (2), we can write the obtained generalized solutions of the nonlinear kink- or soliton-type problem

$$\Omega_L\Big(-\frac{i\varepsilon}{\beta}\frac{\partial}{\partial x}, t, \varepsilon\Big)\psi\Big|_{x=\varphi} = 0, \tag{2}$$

where $\Omega_L(z)$ is the "symbol" of the nonlinear equation. Since the trajectory $x = \varphi(t)$ is apriori unknown in advance, from (3) we obtain

$$\Omega_L\Big(-\frac{i\varepsilon}{\beta}\frac{\partial}{\partial x}, t, \varepsilon\Big)\psi = 0 \quad \text{or} \quad \Omega_L\Big(\frac{\varepsilon p}{\beta}, t, \varepsilon\Big) = 0, \quad p = -i\frac{\partial}{\partial x}.$$

Usual asymptotics w.r.t. $\varepsilon$ of this expression implies the momentum decomposition whose leading terms are used in the weak asymptotics method and the unusual constructions of definitions of generalized solutions used in this method. The relation $\Omega_L(\frac{i\varepsilon p}{\beta}, t, 0) = 0$ gives precisely the standard equation. Similar facts hold for the case of interaction of solitary waves and permit justifying the approach used in the weak asymptotics method [1].

This research is supported by the Russian Foundation for Basic Research (under grant no. 05-01-00912.

**Reference**

[1] V. G. Danilov, G. A. Omel'yanov, and V. M. Shelkovich, *Weak asymptotics method and interaction of nonlinear waves*, Amer. Math. Soc. Transl. (2), **208** (2003), 33–164.

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Shantanu Dave (University of Vienna)

**Global generalized functions on compact manifolds**

We consider distributions on a closed compact manifold as maps on smoothing operators and for any subgroup of the diffeomorphism group we produce a "full type Colombeau algebra" and an equivariant embedding of distributions into this algebra. In particular we shall provide examples for the Symplectic group as well as for the Heisenberg group and $Sp(2n)$.

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Maarten de Hoop (Purdue University)

**To be announced**

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Antoine Delcroix (Université Antilles-Guyane)

**Kernel theorems in spaces of tempered generalized functions**

In analogy to the classical isomorphism between $L(S(R^n), S'(R^m))$ and $S'(R^{n+m})$, we show that a large class of moderate linear mappings acting between the space $G_S(R^n)$ of Colombeau rapidly decreasing generalized functions and the space $G_\tau(R^n)$ of temperate ones admits generalized integral representations, with kernels belonging to $G_\tau(R^{n+m})$. This result contains the classical one.

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Plamen Djakov (Sofia University)

**Spectral gap asymptotics of Schrödinger operators with singular periodic potentials**

[joint work with Boris Mityagin (Ohio)]

Consider the Schrödinger operator $L(v)y = -y'' + v(x)y$ with a $\pi$–periodic potential $v \in H^{-1}_{loc}(\mathbb{R})$. Every such potential has the form $v = const + Q'$, where $Q$ is a $\pi$–periodic $L^2_{loc}(\mathbb{R})$–function, so one may use quasi–derivative $y^{[1]} = y' - Qy$ (following A. Savchuk–A. Shkalikov, R. Hryniv–Ya. Mykytyuk and others) to define properly the Schrödinger operator $L(v)$, and the periodic and anti–periodic Hill–Schrödinger operators $L_{Per^\pm}$ that arise from the same differential expression when it is considered on the interval $[0,\pi]$ with periodic or antiperiodic boundary conditions $Per^\pm: \; y(\pi) = \pm y(0), \;\; y^{[1]}(\pi) = \pm y^{[1]}(0)$. If $v$ is real–valued, then the spectrum of $L(v))$ is absolutely continuous and has a band–gap structure, i.e., it is a union of closed intervals separated by *spectral gaps* $(-\infty, \lambda_0), \; (\lambda_1^-, \lambda_1^+), \; (\lambda_2^-, \lambda_2^+), \cdots, (\lambda_n^-, \lambda_n^+), \cdots$. Moreover, the points $(\lambda_n^\pm)$ are defined by the spectra of the periodic (for even $n$) and anti–periodic (for odd $n$) Hill–Schrödinger operators $L_{Per^\pm}$.

Using the Fourier method developed in [DM07] and following the general scheme of [DM06], we study the relationship between the rate of decay of the spectral gap sequence $\gamma_n = \lambda_n^+ - \lambda_n^-$ and the smoothness of $v$ for real–valued $v$. In the non–self–adjoint case (i.e., for complex–valued potentials $v$) we consider the asymptotics of both $\gamma_n$ and the deviations $\mu_n - (\lambda_n^+ + \lambda_n^-)/2$ (where $\mu_n$ are the eigenvalues of the Hill–Schrödinger operator with Dirichlet boundary conditions $y(\pi) = y(0) = 0$).

**References**

[DM06] P. Djakov and B. Mityagin, *Instability zones of periodic 1D Schrödinger and Dirac operators*, Uspehi Mat. Nauk **61** (2006), 77–182, English text in Russian Math. Surveys **61** (2006).

[DM07] P. Djakov and B. Mityagin, *Fourier method for periodic 1D Schrödinger operators with singular potentials*, preprint, Summer 2007, 1–40.

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Paweł Domański (Adam Mickiewicz University)

**Analytic dependence on parameters of solutions of linear partial differential equations on spaces of distributions**

A surjective linear partial differential operator $P(D) : \mathscr{D}'(\Omega) \to \mathscr{D}'(\Omega)$, $\Omega \subseteq \mathbb{R}^d$, satisfies *real analytic parameter dependence of solutions (RAPD)* if for every family $(f_\lambda) \subseteq \mathscr{D}'(\Omega)$ depending real analytically on $\lambda \in U \subseteq \mathbb{R}^n$ there is a family $(u_\lambda) \subseteq \mathscr{D}'(\Omega)$ also depending analytically on $\lambda$ such that

$$P(D)u_\lambda = f_\lambda \text{ for every } \lambda \in \mathrm{U}.$$

We prove that if $\Omega$ is convex and $P(D)$ is a homogenenous linear partial differential operator with constant coefficients then $P(D)$ satisfies RAPD if and only if $P(D)$ has a continuous linear right inverse. The analogous theorem holds for Beurling type ultradistributions $\mathscr{D}'_{(\omega)}(\Omega)$ instead of distributions. These results are completely different than in the case of smooth or holomorphic dependence.

In order to prove the result we show first that RAPD for $P(D)$ implies some structural property of the kernel $\ker P(D)$ which is close to the classical Vogt's $(\Omega)$ type linear-topological invariant. The introduced invariant, in turn, can be translated via Fundamental Principle into some conditions on plurisubharmonic functions on the zero variety of the polynomial $P$. We use developed by Bonet and the author general theory of surjectivity of completed tensor products of linear operators on spaces of distributions (see, for instance, [BD1], [BD2]). We also use the characterization of $P(D)$ admitting a continuous linear right inverse due to Meise, Taylor and Vogt [MTV].

**References**

[BD1] J. Bonet, P. Domański, *Parameter dependence of solutions of differential equations on spaces of distributions and the splitting of short exact sequences*, J. Funct. Anal. **230** (2006), 329–381.

[BD2] J. Bonet, P. Domański, *The splitting of exact sequences of PLS-spaces and smooth depependence of solutions of linear partial differential equations*, 22 pp., preprint 2006.

[MTV] R. Meise, B. A. Taylor, D. Vogt, *Characterization of the linear partial differential operators with constant coefficients that admit a continuous linear right inverse*, Ann. Inst. Fourier, Grenoble **40** (1990), 619–655.

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AHMED M. A. EL-SAYED (Alexandria University)

**On some problems in the fractional calculus**

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RICARDO ESTRADA (Louisiana State University)

**Distributions that are functions**

It is well-known that any locally Lebesgue integrable function generates a regular distribution, a so-called regular distribution. It is also well-known that many non integrable functions can be regularized to give distributions, but in general not in a unique fashion.

What is not so well-known is that to many distributions one can associate an ordinary function, the function that assigns the distributional point value of the distribution at each point where the value exists, and that in many cases this ordinary function determines the distribution in a unique fashion.

In this talk we consider several classes of distributions that are given in terms of the ordinary function of their point values. In particular we consider distributions that have a point value everywhere, those that have lateral limits at each point, and then introduce the class of distributionally integrable functions. We study several properties of distributions of these classes and apply these ideas to study the boundary behavior of solutions of partial differential equations.

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CLAUDIA GARETTO (University of Innsbruck)

**Closed graph and open mapping theorems for topological $\widetilde{\mathbb{C}}$-modules and applications**

We present closed graph and open mapping theorems for $\widetilde{\mathbb{C}}$-linear maps acting between suitable classes of topological and locally convex topological $\widetilde{\mathbb{C}}$-modules. We give applications of the previous theorems to Colombeau theory as well to the theory of Banach $\widetilde{\mathbb{C}}$-modules. In particular we obtain a necessary condition for $Ginf$-hypoellipticity on the symbol of a partial differential operator with generalized constant coefficients.

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Rudolf Gorenflo (Berlin Free University)

**Distributed order pseudo-differential equations: Cauchy and multi-point problems**

We discuss questions of existence, uniqueness and construction of solutions to Cauchy and multi-point problems for general linear evolution equations with temporal (in general) fractional derivatives (of Caputo-Dzherbashyan type) with distributed orders. Such equations are enjoying increasing interest among researchers in visco-elasticity and in anomalous diffusion, and there are numerical analysts who feel them highly challenging. We find it desirable to put their theory on a strong and general mathematical basis. After an outline of relevant function spaces and their duality structures we treat, by Fourier-Laplace techniques, first the Cauchy problem, then a general multi-point problem (where the values of linear combinations of the unknown solution at different time-instants are prescribed). We condense our results in theorems on strong and on weak solutions. This lecture is mainly based on joint work of the speaker with his co-authors of the following references:

**References**

[1] R. Gorenflo, Yu. Luchko and S. Umarov, *The Cauchy and multi-point partial pseudo-differential equations of fractional order*, Fractional Calculus and Applied Analysis **3** (2000), 249-275.

[2] S. R. Umarov and R. Gorenflo, *Cauchy and nonlocal multi-point problems for distributed order pseudo-differential equations. Part One.*, Zeitschrift fr Analysis und ihre Anwendungen (Journal for Analysis and its Applications) **24** (2005), 449-466.

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Todor Gramchev (Universita di Cagliari )

**Scales of anisotropic spaces of ultradifferentiable functions**

We construct scales of Banach spaces of anisotropic ultradifferentiable functions. We investigate structural properties and geometric features of such spaces and propose applications for solvability of classes of some degenerate partial differential equations.

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Katarzyna Grasela (Cracow University of Technology)

**The algebra of polynomials on the space of ultradifferentiable functions**

By ultradistributions we mean the elements in the dual space of a non-quasi analytic class of infinitely differentiable functions equipped with a natural locally convex topology, as defined by Roumieu and Beurling. We introduce the classs of polynomial ultradistributions, namely the elements of the space $\mathcal{P}(D_M)$, where $D_M$ denotes the space of ultradifferentiable functions in the sense of Roumieu, and described by Komatsu.

The space $\mathcal{P}(D_M)$ contains the space of ultradistributions as a proper subspace and it is the smallest space in which multiplication of ultradistributions is possible. We shall desribe the space $\mathcal{P}(D_M)$ in the terms of the direct sums of symmetric tensor powers of the space $D'_M$. It will also be proved that $\mathcal{P}(D_M)$ is topologically isomorphic to the space $S'$ of ultradistributions for functions in infinitely many variables, which means that it can itself be treated as the space of ultradistributions.

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Michael Grosser (University of Vienna)

**Tensor valued Colombeau functions on manifolds**

A full algebra of generalized (real resp. complex valued) Colombeau functions on smooth manifolds has already been developed by M. Grosser, M. Kunzinger, R. Steinbauer, and J. Vickers. It permits a canonical embedding of distributions and a Lie derivative. For the tensor valued case, the famous impossibility result of Schwartz forbids a componentwise construction based on the scalar case above. Rather, the basic space of (representatives of) regularized objects has to be enriched in a particular way, taking into account that a prescription is needed for transporting tensors from one point of the manifold to another. This transport procedure is necessary for forming averages when regularizing and embedding distributions and for defining flows of vector fields and Lie derivatives. In the talk, a suitable conceptual frame for tensor valued Colombeau functions on manifolds is presented.

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Vladimir Grushevsky (Belorussian State University)

**On non-autonomous differential equations containing the product of generalized functions in the algebra of mnemo-functions**

In our report we consider the following Cauchy problem

$$\begin{cases} \dot{X}(t) = f(t, X(t))\dot{L}(t), \\ X(0) = x_0, \end{cases} \tag{1}$$

where $f : [0, \alpha] \times \mathbb{R} \to \mathbb{R} -$ some discontinuous function, $\dot{L}-$ generalized derivative of the function of finite variation. This equation contains the product of generalized functions hence it's ill-defined problem. To overcome difficulty this equation is considered in the algebra of mnemofunctions. It is shown, that under some conditions the associated solution coincides with the solution of the equation (1) understood in the sense of differential inclusions.

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Maximilian Hasler (University Antilles-Guyane)

**Asymptotic extensions of topological algebras**

We give a general framework for asymptotic extensions of topological modules and algebras, of which Colombeau type algebras are a special case. The construction is very natural and canonical in the sense that the only input is the asymptotic scale and the topology of the "base space", and therefore applies without modification to projective and inductive limit spaces (such as untradistributions and hyperfunctions) and even spaces of generalized functions. A main motivation comes from the fact that Colombeau type spaces are not topological vector spaces but only topological modules: Our framework allows to study possible embeddings of the topological dual of spaces of generalized functions into larger algebras, in the same way as it is done for Schwartz distributions. The present construction gives "for free" several sheaf-theoretical results known only for special cases in the classical theory.

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Andrzej Kamiński (University of Rzeszów)
**On the convolution, compatibility of supports and convolution algebras of generalized functions**
[joint work with S. Mincheva-Kamińska]

We present and discuss new aspects of the classical results concerning the convolution of functions, distributions and other generalized functions of various classes.

To describe fully the situation where the convolution in a given space exists without restrictions on the growth of (generalized) functions we give characterizations in terms of various types of compatibility of their supports. This leads to the construction of convolution algebras of (generalized) functions with supports contained in a given set which is respectively compatible with itself. This general construction embraces the known cases of convolution algebras of (generalized) functions with supports contained in $\mathbb{R}^d_+$ and, more generally, in an acute cone in $\mathbb{R}^d$ (see e.g. V.S. Vladimirov "General Functions in Mathematical Physics", Moscow 1979).

We are going to demonstrate new classes of convolution algebras of (generalized) functions, unknown in the literature, and discuss their properties.

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Anna Karczewska (University of Zielona Góra)
**Regularity of solutions to stochastic Volterra equations**

We consider two classes of linear Volterra equations: of convolution type on $\mathbb{R}^d$ and integro-differential with infinite delay on d-dimensional torus, both driven by spatially homogeneous Wiener process. First, we study existence of solutions to these equations in the space of tempered distributions and then derive conditions under which the solutions take values in Sobolev spaces. We give necessary and sufficient conditions providing regularity of solutions to equations considered. The harmonic analysis techniques and stochastic integration in function spaces are used.

**References**

[1] A. Karczewska, *On the limit measure to stochastic Volterra equations*, J. Integral Equations Appl. (2003).

[2] A. Karczewska, C. Lizama, *Regularity of solutions to stochastic Volterra equations with infinite delay*, Proc. Amer. Math. Soc. (2007).

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Jaewoong Kim (Seoul National University)

**The invariant subspace problem**

If $T \in L(H)$ then $T$ is said to have a nontrivial invariant subspace if there is a subspace $\mathfrak{M}$ of $H$ such that $\{0\} \neq \mathfrak{M} \neq H$ and $T\mathfrak{M} \subset \mathfrak{M}$. In this case we can represent $T$ as

$$T = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \quad \text{on } \mathfrak{M} \oplus \mathfrak{M}^{\perp}. \tag{3}$$

In 1932, J. von Neumann addressed the following problem:

*If $\mathcal{X}$ is a Banach space of dim $\mathcal{X} \geq 2$, does $T \in L(\mathcal{X})$ have a nontrivial invariant subspace?*

Today this is known as the *Invariant Subspace Problem*. In 1984, C.J. Read has shown that there exists an operator acting on $\ell_1$ which has no nontrivial invariant subspace. However the invariant subspace problem remains still open for the cases of separable Hilbert spaces.

In this paper we show:

**Theorem 1**. Let $T \in L(H)$ be an operator such that $||p(T)|| \leq ||p||_{\sigma(T)}$ for every polynomial $p$. If int $\widehat{\sigma(T)}$ is a union of $C^2$-Caratheodory domains then $T$ has a nontrivial invariant subspace. (Here $\widehat{K}$ means the polynomially convex hull of $K$.)

**Corollary 1**. If $T \in L(H)$ is a hyponormal operator such that int $\widehat{\sigma(T)}$ is a boundary of a $C^2$-Caratheodory domain then $T$ has a nontrivial invariant subspace.

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Jong-Ho Kim (NIMS, Daejeon)

**Eigenvalue problems for the -Laplacian and the largest eigenvalue on nonlinear networks**

I talk nonlinear eigenvalue problems for the discrete $p$-Laplace operator subject to different kinds of boundary conditions. I first show the existence of nontrivial solutions to various nonlinear eigenvalue problems. Moreover, I present that the largest eigenvalue for the $p$-Laplacian is exactly $2^{p-1}$.

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JAN KISYŃSKI (Polish Academy of Sciences)

**Regular initial values for linear Cauchy problem in a Banach space**

Let $X$ be a complex Banach space and $A$ a closed linear operator from $X$ to $X$ with domain $D(A)$. It will be assumed that there exist constants $a, b, k, C \in \mathbb{R}_+$ and a continuous function $w : \mathbb{R} \to [1, \infty[$ such that $w(0) = 1$, $\sup \log |x| / \log w(x) < \infty$, $\int_{\mathbb{R}} (1 + x^2)^{-1} w(x)\, dx < \infty$, the set

$$\Lambda_{a,b,w} := \{\lambda \in \mathbb{C} : \Re\lambda \geq a \log w(-Im\lambda) + b\}$$

is contained in the resolvent set of $A$, and $\|(\lambda - A)^{-1}\| \leq Cw(|\lambda|)^k$ for $\lambda \in \Lambda_{a,b,w}$ . Furthermore, it will be assumed that either (a) $w$ is submultiplicative on $\mathbb{R}$ or (b) $w$ is even and $w_{|\mathbb{R}+}$ is non-decreasing. These assumptions imply that $A$ is the generator of an $L(X)$-valued ultradistribution semigroup $S$ for which the space of infinitely differentiable test functions with compact support is the Beurling space $D_w$ in case (a), and some of the spaces of Cioranescu and Zsido in case (b). In particular $w(x) \equiv 1 + |x|$ corresponds to $C_0^\infty(\mathbb{R})$ and $w(x) \equiv \exp\{|x|^{1/s}\}$, $s = const > 1$, to the Gevrey-Beurling space $D_{(s)}(\mathbb{R})$. The lecture will be devoted to relations between the sets $D(A^\infty) := \cap D(A^n)$,

$$\mathbf{J}_0 := lin\, \{S(j)x : x \in X,\, j - \text{ultradifferentiable, supp}\, j \in \mathbb{R}_+\}$$

and the set $X^\infty(A)$ which consists of those elements of $D(A^\infty)$ for which the Cauchy problem $dx(t)/dt = Ax(t)$ for $t \in \mathbb{R}_+, x(0) = x_0$, has a solution $x(\cdot) \in C^\infty(\mathbb{R}_+, X)$.

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IRINA KMIT (Ukrainian Academy of Sciences)

**Correctness of boundary value problems for multidimensional first-order hyperbolic systems**

Extending the classical method of characteristics for one-dimensional hyperbolic systems, we investigate correctness of initial and initial-boundary local and nonlocal problems for a class of multidimensional hyperbolic systems. It is known that one-dimensional semilinear $n \times n$ hyperbolic systems have diagonal form representation (which is crucial for the classical method of characteristics) where the main part represents exactly $n$ directions called characteristics (the main part of each equation corresponds to one unknown and one direction). This contrasts with the case of $m$-dimensional ($m \geq 2$) ($n \times n$)-systems, where a diagonal representation is possible only in very special cases. Thus in multidimensional case we have $n$ equations with $n$ unknowns representing $n^2$ directions in general. By this reason obtaining an integral representation of the boundary problems for such systems becomes a nontrivial task. In spite of this complication we give integral representations of the problems under consideration. This allows us to prove that the problems are correctly posed in a classical sense and, if singular initial data are allowed, to obtain such a result also for generalized solutions. Our analysis covers the case of semilinear (not necessarily symmetric) hyperbolic systems with non-Lipschitz nonlinearities.

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Masaharu Kobayashi (Tokyo Univ. of Science Kagurazaka)
**Modulation spaces $M^{p,q}$ for $0 < p, q \leq \infty$**

The modulation spaces $M^{p,q}$ are one of the function spaces introduced by H.G. Feichtinger in 1980's and are considered as suitable spaces in the analysis of pseudo-differential operators and partial-differential equations in recent years. However they are restricted to the case $1 \leq p, q \leq \infty$. In this talk, we construct the modulation spaces for the range of indexes $0 < p, q \leq \infty$, which coincide with the usual modulation spaces when $1 \leq p, q \leq \infty$ and study their basic properties and their applications.

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Hikosaburo Komatsu (University of Tokyo)
**Heaviside's theory interpreted by Laplace ultradistributions**

As Schwartz wrote in his book, Heaviside's operational calculus is one of the origins of generalized functions. He thus obtained an analytic expression of signal transmission on uniform submarine cables by 1887, that is his biggest contribution in engineering. However, he could not publish the details then, and later again by the intervention of his opponents. His proof published in 1888 does not look like the original. We try to reconstruct his calculation from his descriptions in "Electromagnetic Theory", vol. II , published in 1899, and give our interpretations.

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Sanja Konjik (Novi Sad), M. Kunzinger (Vienna) and M. Oberguggenberger (Innsbruck)

**Generalized calculus of variations and Noether's theorem**

The calculus of variations is a central field of mathematical analysis with numerous applications in such diverse fields as optimal control theory, operations research, stochastic optimization, mechanics, physics, engineering, etc. For variational problems involving singularities, algebras of generalized functions (in the sense of J.F. Colombeau) offer new possibilities beyond the smooth or distributional framework.

In this talk we report on some recent results in this generalized calculus of variations. More precisely, we discuss necessary and sufficient conditions for solutions of generalized variational problems and present a version of Nöther's theorem in the generalized setting.

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Michael Kunzinger (University of Vienna)

**Recent progress in special Colombeau algebras - geometry, topology, and algebra**

Over the past few years there has been considerable progress in the structural understanding of special (or simplified) Colombeau algebras. We report on some main trends in this development: nonsmooth differential geometry, locally convex theory of modules over the ring of generalized numbers, and algebraic aspects of Colombeau theory. Some open problems and directions of future research will be outlined.

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Michael Langenbruch (University of Oldenburg)

**Vector-valued hyperfunctions**

[joint work with P. Domański (Poznań)]

We characterize the locally convex spaces $E$ admitting a reasonable theory of $E-$valued hyperfunctions. Notice that this is always possible for Frechet spaces $E$ by results of Ion and Kawai (Theory of vector-valued hyperfunctions, Publ. Res. Inst. Math. Sci. Kyoto Univ. **11** (1975) 1-19). The duality method and the cohomogical approach to $E-$valued hyperfunctions are considered. As it turns out, our problem is intimately connected to the question when the Laplacian $\Delta : C^\infty(U,E) \to C^\infty(U,E)$ is surjective for any open set $U$. A complete answer is given by means of a dual $(DN)-$type condition when $E$ is a $(DFS)-$space or more generally, when $E$ is a $(PLS)-$space (like spaces of distributions or real analytic functions). The criterion is applied to many concrete spaces from analysis.

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Young-Su Lee (Sogang University)

**Stability of an $n$-dimensional quadratic functional equation in the space of generalized functions**

Making use of the fundamental solution of the heat equation we reformulate and prove the stability theorem of an $n$-dimensional quadratic functional equation in the spaces of generalized functions.

As a main result we prove the following:

Let $u \in \mathcal{D}'$ satisfy the inequality

$$\|u \circ A + \sum_{1 \le i < j \le n} u \circ B_{ij} - n \sum_{i=1}^{n} u \circ P_i\| \le \epsilon,$$

where $A(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$, $B_{ij}(x_1, \cdots, x_n) = x_i - x_j$, and $P_i(x_1, \cdots, x_n) = x_i$. Here $\circ$ means the pullback of generalized functions and the inequality $\|u\| \le \epsilon$ means $|\langle u, \varphi \rangle| \le \epsilon \|\varphi\|_{L^1}$ for all test functions $\varphi$. Then $u$ can be written uniquely in the form

$$u = q(x) + h(x),$$

where $q(x)$ is a quadratic function which satisfies

$$f\left(\sum_{i=1}^{n} x_i\right) + \sum_{1 \le i < j \le n} f(x_i - x_j) = n \sum_{i=1}^{n} f(x_i)$$

and $h(x)$ is a bounded measurable functional such that $\|h\|_{L^\infty} \le (\frac{2}{3} + \frac{n^2+n-4}{n^2+n-2})\epsilon$.

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Oleg Łopuszański (Rzeszów University)
**An operator calculus for generators of $C_0$-semigroups in algebra of polynomials on the space of ultradifferential functions on semiaxis**

On the space $\mathcal{G}_+$ of Gevrey ultradifferential functions with compact supports on semiaxis $[0,\infty)$ we consider the algebra of continuous scalar polynomials $\mathsf{P}(\mathcal{G}_+)$, containing, in particular, the corresponding linear Gevrey ultradistributions $\mathcal{G}'_+$. On algebra of polynomials $\mathsf{P}(\mathcal{G}_+)$ the Fourier transformation extends. Elements of Fourier transform $\widehat{\mathsf{P}}(\mathcal{G}_+)$ are used as symbols for an functional calculus of generators of $C_0$-semigroups, acting in Banach spaces. Represented calculus finds application in problems of the Boson theory.

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Grzegorz Łysik (Polish Academy of Sciences)
**The Mellin transformation of exponentially increasing functions**

The local Mellin transform of a function $u$ continuous on $(0,t]$ can be defined by

$$\mathcal{M}_t u(z) = \int_0^t u(x) x^{-z-1} dx.$$

To make this integral convergent it is usually required that $u$ has polynomial growth at zero i.e. $|u(x)| \le Cx^v$ with some $v \in \mathbb{R}$. Then $\mathcal{M}_t u$ is holomorphic on $\{\Re\, z < v\}$. We shall describe how this definition can be modified in a way suitable for the study of some classes of functions with arbitrary growth at zero. A special attention will be paid on the study of functions defined on the universal covering space of the punctured disc $B(\rho \setminus \{0\})$ bounded by

$$C \exp\{C(|x|^{-k} + |\ln x|^s)\} \quad \text{for} \quad |x| \quad \text{close to zero},$$

where $k > 0$ and $s > 1$. Finally, an application to the study of singular ordinary differential equations will be given.

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Francesco Mainardi (University of Bologna)

**Stochastic processes to model anomalous diffusion in physics**

Following the Grey Noise theory by Schneider we are led to a parametric class of self-similar stochastic processes with stationary increments of Hurst parameter $H \in (0,1)$ (H-sssi) processes). The class of these H-sssi processes depends on two real parameters $\alpha \in (0,2)$ and $\beta \in (0,1]$; we refer to them as generalized grey Brownian motion (ggBm). It includes the fractional Brownian motion (for $\alpha \in (0,2)$, $\beta = 1$) with self-similarity parameter $H = \alpha/2$ and also the "time fractional diffusion processes" (for $\alpha = \beta \in (0,1)$) whose probability density is governed by time fractional diffusion equations, with self-similarity parameter $H = \beta/2$. These processes are consistent with anomalous diffusion in the sense that their variance increases with time following a power law of exponent $\alpha \in (0,2)$. We provide analytical construction and numerical simulation for our stochastic processes.

N.B The work has been carried out with the collaboration of Antonio Mura (PhD Student) and Dr. Gianni Pagnini.

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Jean-André Marti (Univ. Antilles-Guyane)

**Regularity, local and microlocal analysis in theories of generalized functions**

The notion of regularity in algebras or spaces of generalized functions can be formulated in a general way with the help of sheaf theory. This leads to the notion of B-singular support of sections of any presheaf A containing B as subpresheaf.

Many examples are given from some theories of algebras or dual spaces of generalized functions. Some results on propagation of singularities under B-compatible operators permit to explain and summarize the classical results involving differential or pseudo-differential ones.

Now, we give a review on the ideas and results following the Hörmander ones on propagation of singularities and pseudo-differential techniques. They lead to the definition of (frequential) generalized wave front sets for generalized functions or functionals.

Following pseudo-differential characterization of wave front sets, some refined results on propagation of singularities are recalled.

When the generalized regularity is subordinated to an additional condition (as an estimate on the growth of derivatives) characterizing a special property as to belong to an analytic, Gevrey or other class, we can give some corresponding definitions of wave front sets. However, all these kinds of frequential microlocal analysis are based on the Fourier transform which leads to the study of singularities under only linear (including pseudo-differential) operators.

A paradigmatic alternative can be found in the concept of asymptotic analysis for the $(C, E, P)$-algebras. The main advantage is that analysis is compatible with the algebraic stucture of any presheaf F asymptotically associated to $(C, E, P)$-algebras.

Thus, a being a net satisfying some technical condition, the (a,F)-singular singular spectrum inherits good properties with respect to nonlinear operations when $F$ is a presheaf of topological algebras. Moreover, even when $F$ is a presheaf (or sheaf) of vector spaces (like $F = D'$) some results on microlocal analysis are still obtained for nonlinear operations.

We illustrate the results by giving some examples of the propagation of singularities through nonlinear differential operators.

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Eberhard Mayerhofer (University of Vienna)

**Causality in generalized space-times and the dominant energy condition**

We report on a new concept of causality in singular space-times, based on a new generalized point-value description of Lorenzian metrics. We base our considerations on recent developments of generalized pseudo-Riemannian geometry by Kunzinger & Steinbauer. Further the Dominant Energy Conditions for a class of generalized Super Energy Tensors is formulated and shown.

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Irina V. Melnikova (Ural State University)

**Distribution spaces related to abstract stochastic problems**

We consider the stochastic Cauchy problem

$$X'(t) = AX(t) + B\mathbb{W}(t),\ t \in [0, T], \quad u(0) = \xi, \tag{4}$$

with white noise $\mathbb{W}$ and operator $A$ being the generator of a regularized semigroup in a Hilbert space.

At least two reasons that necessitate using distribution spaces for such problems arise while trying to construct a solution to (1) in the natural form $X(t) = U(t)\xi + (U * \mathbb{W})(t),\ t \in [0, T]$, where $U(t)$ are solving operators of the Cauchy problem for the associated homogeneous equation. The first reason is that in the case of $A$ being the generator of a regularized semigroup $\{S(t), t \in [0, \tau)\},\ T < \tau \leq \infty$, operators $U(t)$ are not generally bounded in $H$; instead of them we have the regularized semigroup, that is a regularized solving family $\{S(t)\}$, and we need to define solving operators via this family in a space of distributions. The second one is actually stochastic: we need to define an abstract white noise process.

The talk is concerned with important special cases of regularized semigroups, $k$-convoluted and $R$-semigroups in different distribution spaces.

For the case of $k$-convoluted semigroups we have manage to construct a white noise process and a solution in spaces of abstract ultradistributions. As for the case of $R$-semigroups, in order to have an appropriate space we have introduced spaces of Ivanov type abstract stochastic distributions, where a white noise process is defined and a generalized solution is constructed. (See [1] for background information on spaces of ultradistributions and [2, 3] for abstract ultradistributions and Ivanov spaces of distributions). The work was supported by the RFBR (grant 06-01-00148)

**References**

[1] H. Komatsu, *Ultradistributions I. Structure theorems and characterization*, J. Fac. Sci. Univ. Tokyo. **20** (1973), 25–106.

[2] Irina V. Melnikova and Alexei Filinkov, *The Cauchy problem: Three approaches*, Monographs and Surveys in Pure and Applied Mathematics **120**, CRC Press, London-Washington 2001.

[3] V.K. Ivanov, I.V. Melnikova, *New generalized functions and weak well-posedness of operator problems*, Dokl. Akad. Nauk USSR **317** (1991), 22–26.

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Sławomir Michalik (Stefan Wyszyński University)

**Laplace ultradistributions supported by a cone and the Paley-Wiener type theorem**

We introduce the space of Laplace ultradistributions supported by a convex proper cone. We prove the Paley-Wiener type theorem for this space of Laplace ultradistributions. In particular, we describe the image of this space under the Laplace transformation. As a corollary we obtain the structure theorem for Laplace ultradistributions supported by a cone.

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Piotr Mikusiński (University of Central Florida)

**From Boehmians to pseudoquotients**

Boehmians were introduced in 1981 as a generalizations of regular operators considered by Boehme. The objects are of the form $\left[\frac{f_n}{\varphi_n}\right]$, where the $f_n$'s are functions, $(\varphi_n)$ is an approximate identity, and the exchange property $f_n * \varphi_m = f_m * \varphi_n$ holds. In the first 20 years from the introduction of Boehmians, the work concentrated on applying the construction to different functions $f_n$, different approximate identities, and different interpretations of $*$. In recent years, an abstract generalization of the construction has been studied. The objects have the form of $\left[\frac{x_i}{\varphi_i}\right]$, were the $x_i$'s belong to some set $X$, the $\varphi_i$'s are maps from $X$ to $X$, the $i$'s come from some index set $I$, and the exchange property $\varphi_i x_j = \varphi_j x_i$ is assumed for all $i, j \in I$. The "denominators" need not be approximate identities, but they have to separate points of $X$. This generalization leads to some interesting theoretical problems as well as some new applications, including an extension of the Bochner theorem and the Fourier transform of Lévy measures.

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Svetlana Mincheva-Kamińska (University of Rzeszów)

**On a generalization of the diagonal theorem and its applications in generalized functions**

[joint work with A. Kamiński]

The theorems on infinite matrices being abstract forms of the so-called gliding hump method: the diagonal theorem shown first by J. Mikusiński, its various reformulations given by P. Antosik and their culmination in the form of the basic matrix theorem demonstrated in the book "Matrix Methods in Analysis" by P. Antosik and C. Swartz have proved to be very efficient tools in functional analysis and measure theory. We prove a certain generalization of the mentioned basic matrix theorem which leads to new applications, in particular in the theory of generalized functions.

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Boris Mityagin (Ohio State University)

**Schrödinger and Hill operators with singular $H^{(-1)}$ potentials**

[joint work with Plamen Djakov (Sofia)]

Our goal is to develop a Fourier method for studying the spectral properties (in particular, spectral gap asymptotics) of the Schroedinger operator

$$L(v)y = -y'' + v(x)y, xinR,$$

where $v$ is a singular potential such that

$$v(x) = v(x + \pi), vinH^( - 1)_loc(R).$$

We follow the quasi-derivatives approach which has been successfully used recently by A. Savchuk, A. Shkalikov and R. Hryniv Ya. Mykytyuk.

**References**

[1] *Instability zones of periodic 1D Schroedinger and Dirac operators*, Uspehi Mat. Nauk **61** (2006), 77-182 [English text in Russian Mathematical Surveys, **61** (2006)]

[2] *Fourier method for 1D Schroedinger operators with singular periodic potentials*, preprint, pp. 1-40, Summer 2007.

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Dennis Nemzer (California State University)

**Trigonometric series which vanish on $\sigma < |x| < 2\pi - \sigma$**

Trigonometric series will be discussed in the context of the space of Boehmians. The space of Boehmians is a space of generalized functions whose construction relies on approximate identities and convolution. The first construction appeared in a paper that was published in 1981 [1].

The space of Boehmians is quite large. It contains a subspace which can be identified with the space of Schwartz distributions. There exist Boehmians which are not hyperfunctions and hyperfunctions which are not Boehmians.

We will discuss, among other things, trigonometric series which vanish on $\sigma < |x| < 2\pi - \sigma$, where $0 \leq \sigma < \pi$. Some examples will be given.

**Reference**

[1] J. Mikusiński, P. Mikusiński, *Quotients de suites et leurs applications dans l'analyse fonctionnelle*, C.R. Acad. Sci. Paris Ser. I Math. **293** (1981), 463–464.

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Michael Oberguggenberger (University of Innsbruck)

**Solutions to hyperbolic systems in the dual of the Colombeau algebra**

We study linear hyperbolic systems of partial differential equations with non-smooth, possibly discontinuous coefficients and distributional data. Solutions as elements of the Colombeau algebra have been known to exist since quite some time. However, due to the joint regularization of the data and the coefficients required in this setting, their properties are difficult to investigate.

Based on recent work of Claudia Garetto, we construct solutions in the dual of the Colombeau algebra, the space of sharply continuous linear functionals. This novel approach has the advantage that distributional data can be put into the equation without regularization. In this way, the interplay of the singularities of the coefficients and of the data can be made transparent.

We compute the generalized wave front set of the solution to a transport equation with discontinuous propagation speed and delta functions as initial data. The generalized wave front set turns out to have a more refined and informative structure than the wave front set of the corresponding distributional limit.

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Christian Olivera (Buenos Aires University )

**Neutrix products on $\mathbb{R}^m$ of homogeneous distributions, quasi-associated distributions and quasi-homogeneous distributions**

We study the neutrix products on of homogeneous distributions, quasi-associated distributions and quasi-homogeneous distributions. We obtain some theorems that allow us to evaluate the product of these distributions. The asymptotic expansions in the infinity for distributions, given in [1], are used to get these theorems.

**Reference**

[1] R. Estrada, R.P Kanwal, *A distributional theory for asymptotic expansions. A distributional theory for asymptotic expansions*, Proc. R. Soc. Lond., Ser. A **428** (1990), 399-430.

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LJUBICA OPARNICA (Institute of Mathematics, SANU)

**Semilinear ordinary differential equation coupled with distributed order fractional differential equation**

[joint work with T.M. ATANACKOVIC & S.PILIPOVIĆ (Novi Sad)]

We study the existence and uniqueness for general linear fractional differential equation

$$\sum_{i=1}^{k} a_i D^{\gamma_i} z(t) = g(t), \quad \text{in } \mathscr{S}'_+$$

where $\gamma_i$ are real numbers. Also we consider a system

$$y^{(2)}(t) + z(t) = f(y,t);$$

$$\int_0^2 \phi_1(\gamma) D^\gamma y(t) d\gamma = \int_0^2 \phi_2(\gamma) D^\gamma z(t) d\gamma, \quad t > 0$$

and look for its mild and classical solution. Such systems of ordinary differential equations coupled with distributed order constitutive equations arise in the distributed derivatives models of viscoelasticity and system identification theory.

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IGOR ORLOV (Taurida National University)

**Theorems on permutation of inductive and projective limit and their application to generalized functions.**

For an iterated inductive - projective limit of locally convex spaces

$$\operatorname{proj}\lim_{i\in I} \operatorname{ind}\lim_{j\in J} E_{ij}$$

the equality conditions of iterated limits of the dual spaces

$$\operatorname{proj}\lim_{i} \operatorname{ind}\lim_{j} E^*_{ij} = \operatorname{ind}\lim_{j} \operatorname{proj}\lim_{i} E^*_{ij}.$$

are obtained. The appropriate theorem on nucleus for the case of nuclear spaces is obtained.

The results above are applied to the certain spaces of test functions with inductive -projective topology and to the corresponding spaces of generalized functions.

**Reference**

**1.** Orlov I. V. *Iterated limit theorem for inductively-projective topologies and its application*// Methods of Functional Analysis and Topology **10** (2004), 54-62.

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EMIN OZCAG (Hacettepe University, Ankara)

**Applications of the neutrix calculus to special functions in conjuction with the incomplete Beta and Gamma function**

The incomplete beta function $B_x(a,b)$ is defined by

$$B_x(a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,dt, \qquad a,b>0, \quad 0<x<1.$$

While best known for its applications in Statistics, it is also widely used in many other fields such as actuarial science, economics, finance, survival analysis, life testing and telecommunications.

Its definition can be extended, by regularization, to negative non-integer values of $a$ and $b$.

In this paper, by using the concept of the neutrix due to J. G. van der Corput, we define the incomplete beta function $B_x(a,b)$ for negative integer values of Belorusian St. Univ.$a$ and $b$. Further we prove that the function $\frac{\partial^{m+n}}{\partial a^m \partial b^n} B_x(a,b)$ exists for all $a$ and $b$ and $m,n = 0,1,2,\ldots$. We also define the partial derivatives of $B_x(a,b)$ for negative values of $a$ and $b$.

Finally we consider the incomplete gamma function $\gamma(\alpha,x)$ and its derivatives for negative $\alpha$.

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JAGDISH PANDEY (Carleton University)

**Wavelet expansion of distributions and generalized functions** (Preliminary results)

We construct a mother wavelet which is infinitely differentiable on $\mathbb{R}$ with the support contained in the closed interval [01] and then use it to generate a complete orthonormal systems in the space of square integrable functions defined on $\mathbb{R}$. This result is used to generate a wavelet series expansion of a class of generalized functions interpreting convergence in the weak distributional sense.

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Yea-Hyun Park & Soon Yeong Chung (Sogang University)

**Eigenvalue problem for $(p, w)$-Schrödinger operators on networks**

In this talk, we deal with the $(p, \omega)$-Schrödinger operators

$$\begin{aligned}(-\Delta_{p,\omega} + q(x))u(x) = \quad & -\sum_{y\in V} |u(y) - u(x)|^{p-2}(u(y) - u(x))\frac{\omega(x,y)}{d_w x} \\ & + q(x)|u(x)|^{p-2}u(x), \quad x \in V,\end{aligned}$$

on networks and its eigenvalue problems. After defining the discrete operator $\Delta_{p,\omega}$ on networks, we first discuss the existence of $\lambda \in \mathbb{R}$ which satisfies that the equation

$$-\Delta_{p,w}\phi(x) + q(x)|\phi(x)|^{p-2}\phi(x) = \lambda|\phi(x)|^{p-2}\phi(x)$$

has a nontrivial solution. The $\lambda$ is said to be an eigenvalue of the $(p, \omega)$-Schrödinger operator.

Moreover, we discuss the existence of the solutions of the $(p, \omega)$-Schrödinger equations

$$-\Delta_{p,w}u(x) + q(x)|u(x)|^{p-2}u(x) = f(x)$$

on networks.

**Reference**

[1] J. Garcia-Melian and J. Sabina de Lis, *Maximum and comparison principles for operators involving the p-Laplacian*, J. Math. Anal. Appl. **218** (1998), no. 1, 49–65.

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Dusanka Perisić (University of Novi Sad)

**Structural theorem for generalized Gelafand-Shilov spaces in quasianalytic and nonquasianalytic case**

Test space of generalized Gelafand-Shilov spaces of Beurling-Komatsu and Roumieu-Komatsu type can be identified with the spaces of sequences of ultrapolynomal falloff and their dual spaces with the space of sequences of ultrapolynomial growth. We use the characterizations to give simple proofs of structural theorems for generalized Gelfand-Shilov spaces of Beurling-Komatsu and Roumieu-Komatsu type, both in quasianalytic and nonquasianalytic case.

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Stevan Pilipović (University of Novi Sad)

**Convoluted operator families and abstract Cauchy problems**

Global exponentially bounded convoluted semigroups in Banach spaces are systematically treated with the help of Laplace transform. Especially, we analyze ultradistribution semigroups with non-densely defined generators as well as Fourier-hyperfunction and hyperfunction semigroups and their connections with local integrated $C$-semigroups. Convoluted $C$-cosine functions and semigroups in a Banach space setting extending the classes of fractionally integrated $C$-cosine functions and semigroups are analyzed. Ultradistribution and hyperfunction sines are connected with analytic convoluted semigroups and ultradistribution semigroups. Convoluted semigroups and convoluted cosine functions are considered as mild and classical solutions of the corresponding abstract Cauchy problems.

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Burak Polat (Uludağ University)

**Classical electrodynamics in the sense of distributions**

The role of the theory of generalized functions in complementing Maxwell's field theory is reviewed and developed systematically to display certain aspects of the singular nature of electromagnetic fields in presence of point, curve and surface type nonmaterial and material singularities in the rest frame. We invoke the standard vector operators (gradient, divergence, curl, Laplacian) on the regular and singular components of a scalar/vector generalized function for providing the mathematical tools for an interpretation of the field relations. The singular component of any scalar/vector quantity is assumed to be constructed via the Dirac delta distribution and its directional derivatives of every order. Then these tools are applied to the point and integral form field relations under the general postulation that these relations are valid in the sense of distributions in space-time. Regarding material singularities, spatial jump relations for an anisotropic thin film are derived with special cases of practical interest, on which the influence of geometry (surface curvature) and constitutive parameters can be observed analytically. Results obtained from the Maxwell's relations in integral form rely on the fact that the Green theorems apply in the sense of distributions.

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Yauhen Radyna & Yakov Radyno (Belorusian St. Univ.)

**Generalized functions on the field of p-adic numbers and on the ring of adeles**

In the talk we consider spaces of distributions and mnemofunctions on the field of $p$-adic numbers and on the ring of adeles. The field of $p$-adic numbers is a completion of the rationals with respect to non-archimedian $p$-adic valuation, in the same way as the field of reals is, and the ring of adeles brings all the completions together.

We also consider a number of specials distributions and mnemofunctions which are of particular interest. Among them homogeneous Tate Rs distributions and their generalizations. Fourier analysis is built for distributions and mnemofunctions.

We outline the specifics and current state of the research in the area.

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Dusan Rakić (University of Novi Sad)

**Homogeneous distributions in $\mathcal{D}'_{L^p}$**

[joint work with S. Pilipović & N. Teofanov]

We study the homogeneity property on a scale of subspaces $\mathcal{D}'_{L^q}(\mathbb{R})$, $1 \leq q \leq \infty$ of the space of tempered distributions $\mathcal{S}'(\mathbb{R})$. It is shown that a homogeneous distributions belongs to $\mathcal{D}'_{L^q}(\mathbb{R}^n)$ if and only if its degree of homogeneity belongs to $(-\infty, -\frac{n}{q})$, $1 \leq q \leq \infty$ (if $q = \infty$ then $\frac{n}{q} = 0$).

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Luigi Rodino (University of Torino)

**Gelfand-Shilov spaces and applications**

We consider the Gelfand-Shilov spaces, subspaces of the Schwartz class $S$. Their duals define ultra-distribution spaces containing the tempered distributions $S'$ of Schwartz. We present some applications to partial differential equations. Namely, relevant linear and non-linear equations in Quantum Physics and Applied Mathematics possess global bounded solutions in the Euclidean space, which have Gelfand-Shilov regularity (results in collaboration with M. Cappiello and T. Gramchev). Other applications concern Time-Frequency Analysis; more precisely, taking Gelfand-Shilov functions as window functions, one defines localization operators with ultra-distribution symbol, with boundedness properties on the Lebesgue spaces (results in collaboration with E. Cordero, S. Pilipovic and N. Teofanov).

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Dimitris Scarpalézos (University of Paris 7)

**Analytic and real analytic generalized functions**

Foundation of algebras $\mathcal{G}_H(\Omega), \Omega \subset \mathbb{C}^p$, and $\mathcal{G}_A(\omega),\ \omega \subset \mathbb{R}^d$, of analytic and real analytic generalized functions will be presented. It will be shown that such generalized functions are equal to zero if they are equal to zero at every classical point of a sufficiently rich set. Generalized analytic wave front of $f \in \mathcal{G}(\omega)$ will be analyzed and it will be shown that for a distribution its wave front coincides with the generalized wave front of embedded distribution in $\mathcal{G}(\omega)$. Also the notions of $S$-analyticity and the analyticity will be discussed.

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Vladimir Shelkovich (St. Petersburg St. Univ. of Archit.)

**Strong singular solutions to systems of conservation laws: shocks, $\delta$-, $\delta'$-, and $\delta^{(n)}$-shocks ($n = 2, 3, \ldots$)**

[to be delivered during the Conference]

We consider and discuss the following problems: the definitions of singular solutions (in the form of integral identities) to systems of conservation laws such as shocks, $\delta$-, $\delta'$-, and $\delta^{(n)}$-shocks ($n = 2, 3, \ldots$); the Rankine–Hugoniot conditions for them; geometrical and algebraical aspects of such singular solutions; solving the Cauchy problems admitting such type solutions [1]–[3], [5], [6].

Roughly speaking, $\delta^{(n)}$-shock is a solution to *quasi-linear* system of conservation laws such that for $t > 0$ its components may contain a linear combination *Dirac measures and their derivatives from* 1*-th to* $n$*-th orders.*

We consider $\delta$-shock problems for the one-dimensional systems

$$u_t + \Big(F(u,v)\Big)_x = 0, \quad v_t + \Big(G(u,v)\Big)_x = 0,$$

$$v_t + (G(u,v))_x = 0, \quad (uv)_t + (H(u,v))_x = 0,$$

(where $F(u,v)$, $G(u,v)$, $H(u,v)$ are smooth functions *linear* with respect to $v$), and for multidimensional zero-pressure gas dynamics system. $\delta'$-Shock problems are considered for the one-dimensional system

$$u_t+(f(u))_x = 0, \quad v_t+(f_1(u)v)_x = 0, \quad w_t+(f_{22}(u)v^2+f_{21}(u)w)_x = 0,$$

where $f(u)$, $f_1(u)$, $f_{21}(u)$, $f_{22}(u)$ are smooth functions.

These new results show that systems of conservation laws can develop not only Dirac measures (as in the case of $\delta$-shocks) but their derivatives as well, which gives a new perspective in the theory of singular solutions to systems of conservation laws.

We compare our approach to singular solutions with the Colombeau theory approach [4].

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Vladimir Shelkovich (St. Petersburg St. Univ. of Archit.)
**$p$-Adic wavelets and their application to $p$-adic pseudo-differential equations**
[the subject of this abstract may be discussed with the author]

We study some problems related with the theory of multidimensional $p$-adic wavelets in connection with the theory of multidimensional $p$-adic pseudo-differential operators (in the $p$-adic Lizorkin space) and pseudo-differential equations [1]–[3].

We introduce a new class of multidimensional $p$-adic compactly supported wavelets. These wavelets form an orthonormal complete basis in $L^2(\mathbf{Q}_p^n)$. A criterion for a multidimensional $p$-adic wavelet to be an eigenfunction for a pseudo-differential operator is derived. We prove that these wavelets are eigenfunctions of the Taibleson fractional operator. Since many $p$-adic models use pseudo-differential operators (fractional operator), these results can be intensively used in applications.

Using our theory of $p$-adic pseudo-differential operators on the Lizorkin space [1], [2] and $p$-adic wavelets theory [3], we first develop a new approach to the construction of solutions for a wide class of $p$-adic *linear and semi-linear evolutionary pseudo-differential equations* (in multidimensional case).

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KRYSTYNA SKÓRNIK (Polish Academy of Sciences)

**Professor Jan Mikusiński – the 20th anniversary of his death**

The talk will be presented as a multimedia presentation substantiated with pictures. It covers the following topics connected with Jan Mikusiński: Ancestors:

a) grandparents

b) parents.

1. Jan Stefan Geniusz Mikusiński – academic career
2. Professor Mikusiński in Silesia.
3. Research interests of Jan Mikusiński.
4. International cooperation. Organization activity.
5. Family of Jan Mikusiński.

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ALEXANDER SMIRNOV (Lebedev Physical Institute)

**Fourier transformation of Sato's hyperfunctions**

A new generalized function space in which all Gelfand–Shilov classes $S'^{0}_{\alpha}$ $(\alpha > 1)$ of analytic functionals are embedded is introduced. This space of *ultrafunctionals* does not possess a natural nontrivial topology and cannot be obtained via duality from any test function space. A canonical isomorphism between the spaces of hyperfunctions and ultrafunctionals on $\mathbb{R}^k$ is constructed that extends the Fourier transformation of Roumieu-type ultradistributions and is naturally interpreted as the Fourier transformation of hyperfunctions. The notion of carrier cone that replaces the notion of support of a generalized function for ultrafunctionals is proposed. A Paley-Wiener-Schwartz-type theorem describing the Laplace transformation of ultrafunctionals carried by proper convex closed cones is obtained and the connection between the Laplace and Fourier transformations is established.

These results were obtained in the paper A.G. Smirnov, *Fourier transformation of Sato's hyperfunctions*, Adv. Math. **196** (2005), 310-345.

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Svetlana Smirnova (Taurida National University)

**Compact ellipsoids and compact extrema of the variation functional**

[joint work with I. Orlov]

It is shown that extrema of variation functionals in Sobolev spaces of Hilbert type possess a special non-local character and realize, as a rule, on a complete enough system of the compact ellipsoids. We stated that compact ellipsoids are universal compact sets in Hilbert space and establish connection between rate of decrease of ellipsoid semiaxis and smoothness of corresponding function class. The concrete examples are considered.

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Marcelo Reicher Soares (Saõ Paulo State University)

**On extension theorems for generalized holomorfic functions**

In this work we present two extension theorems for simplified generalized holomorphic functions. The first one is a version, from among the possible presentations, of the classic Hartogs' extension theorem. In this one we leave from a generalized holomorphic function in an open neighbourhood of the boundary from a limited open, extending it, holomorphically, to a full open. In the second theorem it is obtained a generalized version of a classic result, done independently, in 1943, by S. Bochner and F. Severi. For this theorem, we leave from a function that is generalized holomorphic and has a holomorphic representant, in some sense to be made clear in the exposition, on the boundary of a limited domain, we extend holomorphically that function, for the whole domain. From a certain point of view the second theorem radicalizes the hypothesis from the first and, for that, we must to work on a generalized holomorphic function algebra defined in the close of an open set. We utilize, yet, the kernel of Bochner-Martinelli, as well as a representation of a holomorphic function by means of such kernel.

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Sławomir Sorek (University of Rzeszów)

**On the composition of distributions**

[joint work with P. Antosik and A. Kamiński]

The two-argument irregular operation of composition of distributions is defined and studied. Several results concerning the existence of the composition $g(f)$ of distributions $g$ and $f$ are obtained, e.g. in case $g$ is a continuous function and $f$ is a quasi-measure. By a *quasi-measure* we mean a distribution $f$ such that the Łojasiewicz value $f_L(a)$ of $f$ at a point $a$ exists for almost all $a \in \mathbb{R}$ and the mapping $x \mapsto f_L(x)$, denoted by $f_r$ (denote also $f_s := f - f_r$), is a locally integrable function.

The main result has the following form in one-dimensional case:

**Theorem.** *Assume that $g$ is a continuous function on $\mathbb{R}$ such that the limits $\lim_{x\to+\infty} g(x)/x = \alpha$, $\lim_{x\to-\infty} g(x)/x = \beta$ are finite. Assume that $f$ is a measure with the representation $f = f_r + f_s$ and the measure $f_s$ has the canonical representation $f_s = f_s^+ - f_s^-$. Then the composition $g(f)$ of $g$ and $f$ exists and the following formula holds:*

$$g(f) = g \circ f_r + \alpha f_s^+ - \beta f_s^- .$$

The presented theorems lead to numerous astonishing results and formulas interesting to physicists, for example:

$$\sqrt[r]{\sum_{j=0}^{r} a_j\, \delta^j} = \sqrt[r]{a_0} + \sqrt[r]{a_r}\, \delta,$$

where $a_j$ are non-negative numbers and the symbol $\delta^j$ means the result of the operation of substitution of $\delta$ into the function $h$ defined by $h(x) := x^j$ $(j = 0, 1, \ldots, r)$.

The theorems can be extended to the more general case of distributions defined in open subsets of Euclidean spaces.

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Roland Steinbauer (University of Vienna)

**Towards tensor valued Colombeau functions**

A full algebra of generalized (real resp. complex valued) Colombeau functions on smooth manifolds has already been developed by M. Grosser, M. Kunzinger, R. Steinbauer, and J. Vickers. It permits a canonical embedding of distributions and a Lie derivative. In this talk we review this construction with special emphasis on the point of view that commuting of the embedding of distributions with Lie derivatives actually is a consequence of the diffeomorphism-invariance of the embedding. Then we proceed to discuss some foundational issues of a theory of tensor valued Colombeau functions on manifolds, in particular, some obstacles that are direct consequences of the Schwartz impossibility result and ways to circumvent these.

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Mirjana Stojanović (University of Novi Sad)
**Schrödinger equation with nonlinear potential and singular initial data**

We prove the existence-uniqueness theorems for Schrödinger equation with nonlinear potential and singular initial data in the the Colombeau spaces $\mathcal{G}_{p,q}([0,T) \times \mathbb{R}^n)$, $1 \leq p, q \leq \infty$, $n < 8$.

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Arpad Takači (University of Novi Sad)
**Solving mathematical models of the viscoelastic bar**

We consider mathematical models of the shock between a solid body and a linear viscoelastic bar, resting on a solid or viscoelastic base in the frame of operational calculus. We determine the exact and construct the approximate solutions both for the operational and the original equation. We show that this procedure can be applied also for similar problems with initial values being certain generalized functions.

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Djurdjica Takači (University of Novi Sad)
**Approximate solutions of a class of nonlinear operator differential equations**

We consider a class of nonlinear differential equations in the field of Mikusiń ski operators with operator initial conditions. We construct a sequence of operator functions, analyze its character, and prove its type I convergence, under some assumptions, to the exact solution of the considered problem. Thus each term of the obtained sequence can be treated as the approximate solution of the problem. The general theory is applied to a partial integro–differential equation with appropriate initial conditions.

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Joachim Toft (Växjö University)
**Continuity properties for pseudo-differential operators acting on modulation spaces**

Modulation spaces is a family of Banach spaces containing functions and distributions (ultra-distributions) which was introduced by Feichtinger during the 80ś. The classical modulation space norms are obtained by applying mixed Lebesgue norms on the short-time Fourier transforms (STFT) of the distributions under consideration. In general modulation space norms one applies mixed Lebesgue norms which contains weights as well. Modulation spaces might be, in fields like time-frequency analysis and pseudo-differential calculus, convenient to use to make a detailed study of growth, decay and regularity of distributions. Furthermore, there are rather close relations between modulation spaces, and more well-known spaces like Sobolev/Besov spaces and certain čIassicalšymbol classes in pseudo-differential calculus. In this talk we consider continuity of pseudo-differential operators in background of modulation spaces. This means that the operator symbols are assumed to belong to certain modulation spaces, and that continuity for such operators are considered with respect to modulation spaces. In the case of compact operators acting on modulation spaces of Hilbert type, we make a somewhat detailed study in terms of Schatten-von Neumann classes. The embedding properties between modulation spaces and Sobolev/Besov spaces, can then be used to obtain results of more classical character, i.e. the symbols belong to well-known symbol classes and continuity and for the corresponding pseudo-differential operators are considered in background of Sobolev/Besov spaces.

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Igor Tralle (University of Rzeszów)
**Solution of the Dirichlet and Neumann problems for some nonlinear partial differential equations**

The method for searching of the solution to Dirichlet problem for the nonlinear Poisson equation (NPE) with complex coefficients is proposed. We prove that the Dirichlet problem for NPE on the plane can be solved for almost arbitrary simply connected domain which can be mapped conformally into the unit disc. If the functions which maps this domain conformally into the disc is known, the solution can be expressed in an explicit form derived in the paper. Our approach is based on the observation that there is exist some transformation on the complex plane which interrelates the solution of NPE with the solution of Laplace equation. Thus, in order to solve the Dirichlet problem for NPE, one should solve an appropriate auxiliary boundary value problem for the Laplace equation. This boundary value problem can be easily formulated for the circle; it is Cauchy problem and so that two conditions which constitute Cauchy conditions be compatible with each other, the function which represents first of them should be $\pi$-periodic. This result is generalized to the solution of the Dirichlet problem for NPE for almost arbitrary simply connected domain on the plane. The possibility to search for the generalized solutions to Dirichlet problem is also discussed. The solution to Neumann problem for NPE is also considered. This technique is then applied to the solution of the similar Dirichlet and Neumann problems for some other nonlinear PDEs.

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Vincent Valmorin (University of Antilles - Guyane)

**On Schwartz kernel theorem in algebras of generalized functions**

[to be delivered during the Conference]

A new approach of the generalization of Schwartz's kernel theorem is given explaining the meaning of certain hypotheses in preceding work of this type. In the present work we mainly consider linear maps from algebras of classical functions to algebras of generalized ones. Results using particular properties of the $G^{\infty}$ class are given and a straightforward relation between the classical and the generalized versions of the Schwartz's kernel theorem is established.

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Vincent Valmorin (University of Antilles)

**A global construction of algebras of generalized functions**

[the subject of this abstract may be discussed with the author]

An original method for the construction of algebras of generalized functions is given, which covers both Colombeau simplified algebras and Rosinger algebras cases, the main used such algebras. Examples are given showing how this method works for the most known algebras in this area.

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Hans Vernaeve (Universität Innsbruck)

**Algebra homomorphisms of Colombeau generalized functions**

We show that for smooth manifolds $X$ and $Y$, any algebra isomorphism between the special algebra of Colombeau generalized functions on $X$, resp. $Y$ is given by composition with a unique Colombeau generalized function from $Y$ to $X$. In order to prove this result, we identify the multiplicative linear functionals from the special algebra of Colombeau generalized functions on $X$ to the ring of Colombeau generalized numbers. Up to multiplication with an idempotent generalized number, they are given by an evaluation map at a compactly supported generalized point on $X$.

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James Vickers (University of Southampton)

**Generalized differential geometry**

In this talk we will start by briefly reviewing the main features of Colombeau's theory of generalized functions and show how this may be extended to a manifestly diffeomorphism invariant theory of scalar fields on manifolds. It is then shown that it is not possible to extend this in a simple way to a theory of generalized tensor fields on manifolds by simply embedding componentwise. Instead it is necessary to introduce some additional structure which allows one to compare tensor fields at nearby points on a manifold. This additional structure is provided by transport operators which may be given by specifying some background connection. This enables one to develop a theory of generalized tensor fields on a manifold. There exist canonical embeddings of both smooth and distributional tensor fields and these agree when applied to smooth tensors. The generalized Lie derivative for generalized tensor fields is defined and it is shown that this commutes with the embedding. The generalized covariant derivative is also introduced and it is shown that this commutes with the embedding at the level of association.

The concept of generalized metric (in the full algebra) is introduced and this is used to develop a theory of generalized differential geometry. It is shown that (for example) a continuous metric may be

embedded into the algebra of generalized tensor fields and results in a generalized metric with well defined connection and curvature. Some examples are then given which show how this may be applied to describe spacetimes with weak singularities in terms of nonsingular generalized spacetimes.

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Jasson Vindas (Louisiana State University)

**Structural theorems for quasiasymptotics of Schwartz distributions**

Complete structural theorems for quasiasymptotics of distributions are presented. The cases at infinity and the origin are both analyzed. *Asymptotically and associate asymptotically homogeneous functions* with respect to a slowly varying function are introduced and some of their basic properties are discussed, they are the main tool of this analysis. Further characterizations of quasiasymptotics are also presented. Relations of these structural theorems with a pointwise Fourier inversion formula for Łojasiewicz point values are discussed.

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Dietmar Vogt (University of Wuppertal)

**Partial differential operators in spaces of real analytic functions: right inverses and solutions with parameters**

Let $P(D)$ be a differential polynomial acting on the space $A(\Omega)$ of real analytic functions on an open set $\Omega \subset \mathbb{R}^n$. The problem of characterizing those operators $P(D)$ which admit a continuous linear right inverse in $A(\Omega)$ is still unsolved, even for $\Omega = \mathbb{R}^n$. We present particular solutions, necessary conditions for the general case and sufficient conditions for special cases. The problem is linked to the problem of solving $P(D)$ for real analytic functions with parameters. We discuss the connection and give solutions also for this problem.

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Ursula Westphal (University of Hannover)

**K-functionals related to fractional powers of operators**

In this talk we will discuss some results obtained jointly with W. Trebels. We consider $K$-functionals with respect to a Banach space $X$ and the domains of fractional powers of operators which are infinitesimal generators of equibounded strongly continuous semigroups of operators on $X$. Several characterizations of these $K$-functionals are given, e.g. by truncated hypersingular integrals, which allow immediate applications to approximation theory. As a tool we use a functional calculus due to L. Schwartz based on the class of integrable distributions.

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Man Wah Wong (York University)

**Fourier-Wigner transforms and Liouville's theorems for the sub-Laplacian on the Heisenberg group**

By decomposing the sub-Laplacian on the Heisenberg group into a family of twisted Laplacians parametrized by Planck's constant and using Fourier-Wigner transforms so parametrized, we prove that the twisted Laplacians are globally hypoelliptic in the setting of tempered distributions. This result on global hypoellipticity is then used to obtain Liouville's theorems for harmonic functions for the sub-Laplacian on the Heisenberg group.

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