

GEOMETRY OF JETS AND FIELDS

10-16 MAY 2015, BĘDLEWO, POLAND

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**WARSAW CENTER OF MATHEMATICS
AND COMPUTER SCIENCE**

INVITED SPEAKERS

- Jose-Fernando Carinena (University of Zaragoza)
- Janusz Grabowski (Polish Academy of Sciences)
- Partha Guha (S. N. Bose National Centre for Basic Sciences)
- Simone Gutt (Universite Libre de Bruxelles)
- Alberto Ibort (Universidad Carlos III de Madrid)
- Madeleine Jotz Lean (University of Sheffield)
- Jerzy Kijowski (Centre of Theoretical Physics Polish Academy of Sciences)
- Yvette Kossmann-Schwarzbach (Ecole Polytechnique)
- Manuel de Leon (Instituto de Ciencias Matematicas)
- Charles-Michel Marle (Universite Pierre et Marie Curie)
- Giuseppe Marmo (Universita di Napoli "Federico II")
- Juan-Carlos Marrero (Universidad de La Laguna)
- Eduardo Martinez (Memorial University of Newfoundland)
- Guowu Meng (Hong Kong University of Science and Technology)
- Norbert Poncin (University of Luxemburg)
- Olga Rossi (The University of Ostrava)
- Gennadi Sardanashvily (Moscow State University)
- Yunhe Sheng (School of Mathematics Jilin University)
- Alexandre Vinogradov (Levi-Civita Institute)
- Luca Vitagliano (University of Salerno)
- Aissa Wade (Penn State University)
- Ping Xu (Penn State University)

AIM AND SCOPE

The main topics of the conference are:

- Geometry of multivector and jet bundles.
- Covariant formulation of field theories.
- Infinite order theories: geometry, symmetries, and reductions.
- Variational calculus.
- Supergeometric and cohomological methods in PDEs and field quantization.

PRACTICAL INFORMATION

Up-to-date information about the conference can be found on our website <http://www.impan.pl/~gjf>. In case of any problems you can contact the organizers by email at conference.gjf@gmail.com or directly by phone:

- (+48) 61 813 51 87 Conference Center in Będlewo
- (+48) 604 579 071 Katarzyna Grabowska (in Będlewo since Sunday early afternoon)
- (+48) 516 403 968 Javier de Lucas (in Będlewo since Saturday early afternoon)

ACCOMMODATION AND BOARDING

The conference will take place from 10th to 16th May 2015 in the **Mathematical Research and Conference Center in Będlewo**, Poland. The participation is free of charge. The organizers will cover the full-board accommodation costs of the stay in Będlewo for all the accepted participants. Accompanying persons must cover the costs of their stay, which is approximately 30-35 Euro per day.

TRAVEL

Two conference buses will be organized from Poznań Airport and Poznań Main Station (Poznań Główny) to the conference center. The buses will carry the sign “Geometry of Jets and Fields”.

- **Sunday 10th May**, from the **Main Train Station** bus leaving shortly after **17.30**. The bus will be waiting at the parking place near Dworzec Letni (Summer Train Station). Dworzec Letni is one of the buildings of the main train station in Poznań. One can see it on Google Street View (look for “Dworzec Letni Poznań”).
- **Sunday 10th May**, from **Poznan Airport** bus leaving around **16.30**. The bus will be waiting near the entrance.

Some participants have already been contacted by e-mail about **taxi service**. Meeting point with the taxi driver **at the train station is near ticket office nr 1**. Meeting point with the taxi driver **at the airport is near the money exchange office**. The Conference Center in Będlewo cooperates with the taxi company called “**Taxi Express**”. The travel from Poznań center to Będlewo should cost approximately 50 Euro (200 PLN).

Timetable and organization of **departures** will be prepared during the conference.

SCHEDULE

	11 th May	12 th May	13 th May	14 th May	15 th May
09.00-10.00	Ch. M. Marle	S. Gutt	A. Vinogradov	G. Sardanashvily	L. Vitagliano
10.00-11.00	Y. Kosmann-Schwarzbach	A. Wade	O. Rossi	M. de Leon	P. Guha
11.00-11.30	COFFEE	COFFEE	COFFEE	COFFEE	COFFEE
11.30-12.30	J. Grabowski	N. Poncin	J.F. Carinena	A. Ibert	E. Martinez
13.00-14.00	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14.00-15.00	BREAK	BREAK	TRIP	BREAK	BREAK
15.00-16.00	G. Marmo	M. Jotz Lean		G. Meng	P. Xu
16.00-16.30	COFFEE	COFFEE		COFFEE	COFFEE
16.30-17.30	J. Kijowski	Y. Sheng		J.C. Marrero	E. Vishnyakova C. Vizman
17.30-18.30		poster session		poster session	best poster
19.00	GRILL	DINER	BANQUET	DINER	DINER

SOCIAL EVENTS

- **Monday 11th May** from **19.00**: a special dinner with grill, beer, and bonfire
- **Wednesday 13th May 14.00-18.00**: a trip to Poznań, where we will visit the local brewery
- **Wednesday 13th May** from **19.00**: conference dinner

SCIENTIFIC PROGRAM

Saturday-Sunday 9th – 10th May

all day arrivals, registration

Monday 11th May

- 09.00-10.00 **Charles-Michel Marle**: *The works of William Rowan Hamilton in Geometric Optics and the Malus-Dupin theorem*
- 10.00-11.00 **Yvette Kosmann-Schwarzbach**: *Multiplicativity, from Lie groups to generalized geometry*
- 11.30-12.30 **Janusz Grabowski**: *New developments in geometric mechanics*
- 15.00-16.00 **Giuseppe Marmo**: *A quantum route to Hamilton-Jacobi theory*
- 16.30-17.30 **Jerzy Kijowski**: *Geometry of q-bit from geometric quantization*

Tuesday 12th May

- 09.00-10.00 **Simone Gutt**: *Submanifolds in symplectic geometry and Radon transforms*
- 10.00-11.00 **Aissa Wade**: *On cosymplectic groupoids*
- 11.30-12.30 **Norbert Poncin**: *Multi-graded algebra and geometry*
- 15.00-16.00 **Madeleine Jotz Lean**: *Poisson Lie 2-algebroids, Courant algebroids and the geometrisation of positively graded manifolds of degree 2*
- 16.30-17.30 **Yunhe Sheng**: *Lie 2-algebras, homotopy Poisson manifolds and Courant algebroids*
- 17.30-18.30 poster session

Wednesday 13th May

- 09.00-10.00 **Alexandre Vinogradov**: *DCinCA, NPDEs and (Q)FT : some problems and perspectives*
- 10.00-11.00 **Alberto Ibort**: *On the multisymplectic formalism of first-order Hamiltonian field theories on manifolds with boundary: an application to Palatini's gravity*
- 11.30-12.30 **Jose-Fernando Carinena**: *Revisiting Lie integrability by quadratures from a geometric perspective*
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SCIENTIFIC PROGRAM (CONTINUED)

Thursday 14 th May	
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09.00-10.00	Gennadi Sardanashvily: <i>Noether theorems in a general setting. Reducible graded Lagrangians</i>
10.00-11.00	Manuel de Leon: <i>Hamilton-Jacobi theory in Cauchy data space</i>
11.30-12.30	Olga Rossi: <i>Geometry of PDEs and Hamiltonian systems</i>
15.00-16.00	Guowu Meng: <i>Tulczyjew's approach for particles in gauge fields</i>
16.30-17.30	Juan-Carlos Marrero: <i>Hamilton-Poincaré field equations</i>
17.30-18.30	poster session

Friday 15 th May	
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09.00-10.00	Luca Vitagliano: <i>Vector bundle valued differential forms on NQ-manifolds</i>
10.00-11.00	Partha Guha: <i>Inverse problem of calculus of variations and the last Jacobi multiplier</i>
11.30-12.30	Eduardo Martinez: <i>Jets and fields on Lie algebroids</i>
15.00-16.00	Ping Xu: <i>Infinite jets of exponential maps and L_∞-algebras</i>
16.30-17.00	Elizaveta Vishnyakova: <i>Flag supermanifolds: definition, properties and applications</i>
17.00-17.30	Cornelia Vizman: <i>Central extensions of the Lie algebra of Hamiltonian vector fields</i>
17.30-18.00	best poster talk

PLENARY TALKS

Jose F. Cariñena (University of Zaragoza)

Revisiting Lie integrability by quadratures from a geometric perspective

Abstract: The classical result of Lie on integrability by quadratures will be reviewed and some generalizations will be proposed. After a short review of the classical Lie theorem, a finite dimensional Lie algebra of vector fields is considered and the most general conditions under which the integral curves of one of the fields can be obtained by quadratures in a prescribed way will be discussed, determining also the number of quadratures needed to integrate the system. The theory will be illustrated with examples and an extension of the theorem where the Lie algebras are replaced by some distributions will also be presented.

Janusz Grabowski (Polish Academy of Sciences)

New developments in geometric mechanics

Abstract: We introduce the concept of a graded bundle, generalizing that of a vector bundle, its linearization, and a double structure of this kind. Then, we present applications in classical field theories, including the Plateau problem, and mechanics with higher order Lagrangians.

Partha Guha (S. N. Bose National Centre for Basic Sciences)

Inverse problem of calculus of variations and the last Jacobi multiplier

Abstract: TBA

Simone Gutt (Universite Libre de Bruxelles)

Submanifolds in symplectic geometry and Radon transforms

Abstract: We shall present in the symplectic framework an analogue to the classical space forms and study the spaces of their totally geodesic submanifolds.

Alberto Ibort (Universidad Carlos III de Madrid)

On the multisymplectic formalism of first-order Hamiltonian field theories on manifolds with boundary: an application to Palatini's gravity

Abstract: The multisymplectic formalism for first-order field theories on manifolds with boundary is discussed. A theory of boundary conditions and a canonical formalism near the boundary are obtained. Palatini's gravity will be reviewed from this perspective.

Madeleine Jotz Lean (University of Sheffield)

Poisson Lie 2-algebroids, Courant algebroids and the geometrisation of positively graded manifolds of degree 2

Abstract: D. Roytenberg and P. Severa independently found an equivalence between symplectic Lie 2-algebroids and Courant algebroids, and D. Li-Bland established a correspondence between (Poisson) Lie 2-algebroids and (LA-) VB-Courant algebroids.

In this talk I will present a new manner to retrieve the Courant algebroid corresponding to a given symplectic Lie 2-algebroid. In order to do this, I will describe split Lie 2-algebroids via Dorfman 2-representations, split Poisson manifolds of degree 2 via self-dual representations up to homotopy, and the compatibility conditions for these structures to define a split Poisson Lie 2-algebroid.

In the second part of the talk, I will give a general overview of the equivalences between positively graded manifolds of degree 2 and metric double vector bundles, between Poisson manifolds

of degree 2 and metric VB-algebroids, between Lie 2-algebroids and VB-Courant algebroids, and between Poisson Lie 2-algebroids and LA-Courant algebroids. If time permits, I will illustrate these equivalences with more examples.

Jerzy Kijowski (Centre of Theoretical Physics Polish Academy of Sciences)
Geometry of q-bit from geometric quantization

Abstract: The talk will contain basic notions and results of the Theory of Geometric Quantization. Its applicability and its limitations will be discussed. Finally, a miracle will be presented which I have discovered recently: a possibility to describe quantum mechanics of a spin (e.g. a q-bit) in this framework. This way we "discover" spinors (in a context which is entirely new) and propose a method to analyze problems of quantum informatics by means of a Wigner function for a q-bit.

Yvette Kosmann-Schwarzbach (Ecole Polytechnique)
Multiplicativity, from Lie groups to generalized geometry

Abstract: We survey the concept of multiplicativity, from its initial appearance in the definition of the Poisson Lie groups to the far-reaching generalizations for multivectors and differential forms in the geometry and the generalized geometry of Lie groupoids and Lie algebroids.

Manuel de León (Instituto de Ciencias Matematicas)
Hamilton-Jacobi theory in Cauchy data space

Abstract: We shall develop a Hamilton-Jacobi theory of Classical Field Theories using a multisymplectic setting, as well as the corresponding Hamilton-Jacobi theory in the Cauchy data space

Charles-Michel Marle (Universite Pierre et Marie Curie)
The works of William Rowan Hamilton in Geometric Optics and the Malus-Dupin theorem

Abstract: In this talk I will discuss the works of William Rowan Hamilton in Geometric Optics, with emphasis on the Malus-Dupin theorem. According to that theorem, a family of light rays depending on two parameters can be focused to a single point by an optical instrument made of reflexing or refracting surfaces if and only if, before entering the optical instrument, the family of rays is rectangular (*i.e.*, admits orthogonal surfaces). Moreover, that theorem states that a rectangular system of rays remains rectangular after an arbitrary number of reflexions through, or refractions across, smooth surfaces of arbitrary shape. I will present the original proof of that theorem due to Hamilton, along with another proof founded in symplectic geometry. It was the proof of that theorem which led Hamilton to introduce his *characteristic function* in Optics, then in Dynamics under the name *action integral*.

Giuseppe Marmo (Universita di Napoli "Federico II")
A quantum route to Hamilton-Jacobi theory

Abstract: There is a wide spread belief that the appropriate description of the physical world should be quantum, therefore one should require that the classical description should be an appropriate limit of the quantum one. From this point of view it is quite reasonable to ask about the fate of the complex structure and the linear structure available in the quantum setting but absent at the classical level. In this talk I shall discuss some of these questions, emerging when we go from the linear Schroedinger equation to the nonlinear Hamilton-Jacobi equation.

Juan-Carlos Marrero (Universidad de La Laguna)
Hamilton-Poincaré field equations

Abstract: In this talk, I will present several geometric descriptions of the Hamilton-Poincaré field equations for an equivariant hamiltonian section. First of all, I will discuss a local derivation of these equations. Next, I will present an intrinsic expression of the equations using the theory of prolongations of invariant vector fields to the reduced extended multimomentum bundle. A third description of the Hamilton-Poincaré field equations will be discussed using a principal connection (in the principal bundle with structural group, the symmetry group, and with total space, the configuration space of the theory). Finally, I will apply the previous results to some examples.

Eduardo Martínez (University of Zaragoza)
Jets and fields on Lie algebroids

Abstract: I will review on the extension of the concept of jet to the context of Lie algebroids and its application in Classical Field Theory.

Guowu Meng (Hong Kong University of Science and Technology)
Tulczyjew's approach for particles in gauge fields

Abstract: In this talk, we shall report that, via an idea due to Shlomo Sternberg, Tulczyjew's original approach to the dynamics of an "electrically-neutral" particle works equally well for the dynamics of an "electrically-charged" particle in non-abelian gauge fields.

Norbert Poncin (University of Luxemburg)
Multi-graded algebra and geometry

Abstract: The aim of the talk is to present a generalization of superalgebra and supergeometry to \mathbb{Z}_2^n -gradings, $n > 1$. The corresponding sign rule is not given by the product of the parities, but by the scalar product of the involved \mathbb{Z}_2^n -degrees. This \mathbb{Z}_2^n -supergeometry exhibits interesting differences with classical supergeometry, provides a sharpened viewpoint, and has better categorical properties. Further, it is closely related to Clifford calculus: Clifford algebras have numerous applications in Physics, but the use of \mathbb{Z}_2^n -gradings has never been investigated. If time permits, the \mathbb{Z}_2^n -Berezinian determinant and the corresponding integration theory will be discussed.

Olga Rossi (The University of Ostrava)
Geometry of PDEs and Hamiltonian systems

Abstract: The talk will survey Hamiltonian field theory in jet bundles for first and some second order Lagrangians, and discuss geometric structures associated with Euler–Lagrange and Hamilton equations.

Gennadi Sardanashvily (Moscow State University)
Noether theorems in a general setting. Reducible graded Lagrangians

Abstract: Noether theorems are formulated in a general case of reducible degenerate Grassmann-graded Lagrangian theory of even and odd variables on graded bundles. A problem is that any Euler-Lagrange operator satisfies Noether identities, which therefore must be separated into the trivial and non-trivial ones. These Noether identities can obey first-stage Noether identities, which in turn are subject to the second-stage ones, and so on. Thus, there is a hierarchy of non-trivial Noether and higher-stage Noether identities. This hierarchy is described in homology terms. If a certain homology regularity conditions holds, one can associate to a reducible degenerate Lagrangian the exact

Koszul-Tate chain complex possessing the boundary operator whose nilpotentness is equivalent to all complete non-trivial Noether and higher-stage Noether identities. Since this complex is necessarily Grassmann-graded, Lagrangian theory on graded bundles is considered from the beginning, and is formulated in terms of the Grassmann-graded variational bicomplex. Its cohomology defines a first variational formula whose straightforward corollary is the first Noether theorem. Second Noether theorems associate to the above mentioned Koszul-Tate complex a certain cochain sequence whose ascent operator consists of the gauge and higher-order gauge symmetries of a Lagrangian system. If gauge symmetries are algebraically closed, this ascent operator is prolonged to the nilpotent BRST operator which brings the gauge cochain sequence into a BRST complex, and thus provides a BRST extension of an original Lagrangian system. [G.Sardanashvily, *arXiv*: 1411.2910]

Yunhe Sheng (School of Mathematics Jilin University)

Lie 2-algebras, homotopy Poisson manifolds and Courant algebroids

Abstract: In this talk, we study Maurer-Cartan elements on homotopy Poisson manifolds of degree n , which unify many twisted, or homotopy structures in Poisson geometry and mathematical physics, such as twisted Poisson manifolds, quasi-Poisson \mathfrak{g} -manifolds and twisted Courant algebroids. We prove that the cotangent bundle of a homotopy Poisson manifold of degree n is a symplectic NQ-manifold of degree $n + 1$. Using the fact that the dual of an n -term L_∞ -algebra is a homotopy Poisson manifold of degree $n - 1$, we obtain a Courant algebroid from a 2-term L_∞ -algebra \mathfrak{g} via the degree 2 symplectic NQ-manifold $T^*[2]\mathfrak{g}^*[1]$. Then, we derive a 2-term L_∞ -algebra from a given one. This construction could produce many interesting examples. By integrating the Lie quasi-bialgebroid associated to the Courant algebroid, we obtain a Lie-quasi-Poisson groupoid from a 2-term L_∞ -algebra, which is proposed to be the geometric structure on the dual of a Lie 2-algebra. At last, we obtain an Ikeda-Uchino algebroid from a 3-term L_∞ -algebra.

Alexandre M. Vinogradov (Levi-Civita Institute)

DCinCA, NPDEs and (Q)FT: some problems and perspectives

Abstract: I shall try to present an unifying view on the current "differential mathematics" including classical differential geometry (CDE) and its natural generalization known as modern geometry of NPDEs.

In the first part of my talk I'll sketch how all structures of modern CDE and many others are almost algorithmically deduced from mathematically formalized observability mechanism in classical physics. In particular, it will be shown that the "observability considerations" leads to reveal a purely algebraic nature of differential calculus and hence its logical structure in the form of functors of differential calculus over commutative (in any sense) algebras (DICOCA). Some immediate applications to physics and mechanics will be outlined.

In the second part of the talk I will informally explain how to develop differential calculus and, in particular, differential geometry, on the "manifold of all solutions of a given NPDE". Essentially, this will be a simplified presentation of Secondary Calculus. Also my intention is to formulate some problems and to discuss some perspectives.

Elizaveta Vishnyakova (MPI Bonn)

Flag supermanifolds: definition, properties and applications

Abstract: Yu.I. Manin introduced four series of compact complex homogeneous supermanifolds corresponding to four series of classical linear complex Lie superalgebras: the general linear Lie superalgebra $\mathfrak{gl}_{m|n}(\mathbb{C})$; the orthosymplectic Lie superalgebra $\mathfrak{osp}_{m|2n}(\mathbb{C})$ that annihilates a non-degenerate even symmetric bilinear form; the linear Lie superalgebra $\pi\mathfrak{sp}_{n|n}(\mathbb{C})$ that annihilates a non-degenerate

odd skew-symmetric bilinear form; the linear Lie superalgebra $\mathfrak{q}_{n|n}(\mathbb{C})$ that commutes with an odd involution. These supermanifolds are called *supermanifolds of flags* in Case 1, *supermanifolds of isotropic flags* in Cases 2 and 3, and *supermanifolds of π -symmetric flags* in Case 4. As in the classical case flag supermanifolds are \mathcal{G} -homogeneous, where \mathcal{G} is one of the following supergroups: $GL_{m|n}(\mathbb{C})$, $OSP_{m|2n}(\mathbb{C})$, $PISp_n(\mathbb{C})$ or $Q_n(\mathbb{C})$, respectively.

We will give the notion of a flag supermanifold via functor of points and as a factor of a Lie supergroup modulo a parabolic subgroup. Further, we will discuss several unexpected properties of flag supermanifolds. For example, almost all flag supermanifolds fail to possess an embedding in a projective superspace. Another interesting question here is whether a flag supermanifold has a non-trivial local deformation. If we have time we will give some applications of flag supermanifolds in representation theory of classical Lie superalgebras and in the theory of super Riemann surfaces.

Luca Vitagliano (University of Salerno)
Vector bundle valued differential forms on NQ -manifolds

Abstract: Geometric structures on NQ -manifolds, i.e. non-negatively graded manifolds with an homological vector field, encode non-graded geometric data on Lie algebroids and their higher analogues. A particularly relevant class of structures consists of vector bundle valued differential forms. (Pre-)symplectic forms, (pre-)contact structures and, more generally, distributions are in this class. I describe vector bundle valued differential forms on non-negatively graded manifolds in terms of non-graded geometric data. Moreover, I use this description to present, in a unified way, novel proofs of known results, and completely new results about degree one NQ -manifolds equipped with certain geometric structures, namely symplectic structures, contact structures, (already present in literature) and, more generally, locally conformal symplectic, presymplectic and precontact structures (not yet present in literature). I also discuss the case of generic vector bundle valued higher order forms, in particular multisymplectic structures.

Cornelia Vizman (West University of Timisoara)
Central extensions of the Lie algebra of Hamiltonian vector fields

Abstract: For a connected symplectic manifold, we classify the continuous central extensions of the Poisson Lie algebra, the Lie algebra of Hamiltonian vector fields, and the Lie algebra of symplectic vector fields.

This is a joint work with Bas Janssens from Utrecht University.

Aissa Wade (Penn State University)
On cosymplectic groupoids

Abstract: A cosymplectic groupoid is a Lie groupoid $G \rightrightarrows M$ endowed with a multiplicative cosymplectic structure. It's clear that the base manifold M of any cosymplectic groupoid G is equipped with a Poisson tensor together with a Poisson vector field: these are the push-forward (under the source map s) of the Poisson tensor on G and its Reeb vector field, respectively. Consequently, the 1-jet of bundle J^1M of the base manifold M has a canonical Lie algebroid structure. We will discuss how certain Lie algebroid structures on the 1-jet of bundle of some Poisson manifolds can be integrated into cosymplectic Lie groupoids.

Ping Xu (Penn State University)
Infinite jets of exponential maps and L_∞ -algebras

Abstract: Exponential maps arise naturally in the contexts of Lie theory and connections on smooth manifolds. The infinite jets of these classical exponential maps are related to the Poincaré–Birkhoff–

Witt isomorphism and the complete symbol of differential operators. We will explain how to extend them to a variety of diverse situations including foliations and complex manifolds. In particular, we will show how such maps induce an interesting class of L_∞ -algebras.

POSTER SESSION

Bogdan Balcerzak (Łódź University of Technology)
Dirac operators on anchored vector bundles

Abstract: Dirac type operators on anchored vector bundles with respect to different geometric structures will be defined and discussed.

Andrew James Bruce (Polish Academy of Sciences)
Higher order mechanics on graded bundles

Abstract: We discuss the applications of the recently discovered weighted Lie algebroids to the theory of higher order Lagrangian mechanics on graded bundles following the geometric ideas of Tulczyjew. As a particular example we will focus on higher order mechanics on Lie algebroids, which is motivated by reductions of higher order systems that possess symmetries.

Ioan Bucataru (Alexandru Ioan Cuza University)
(joint work with Oana A. Constantinescu)
Generalized Helmholtz conditions for non-conservative Lagrangian systems

Abstract: In this paper we provide generalized Helmholtz conditions, in terms of a semi-basic 1-form, which characterize when a given system of second order ordinary differential equations is equivalent to the Lagrange equations, for some given arbitrary non-conservative forces. Our formulation allows, in some cases, to study the formal integrability of the proposed generalized Helmholtz conditions. These conditions, when expressed in terms of a multiplier matrix, reduce to those obtained previously by Mestdag, Sarlet and Crampin, for the particular case of dissipative or gyroscopic forces. We provide examples where the proposed generalized Helmholtz conditions, expressed in terms of a semi-basic 1-form, can be integrated and the corresponding Lagrangian and Lagrange equations can be found.

Ogul Esen (Yeditepe University)
(joint work with Hasan Gümrül (Australian College of Kuwait))
Tulczyjew's triplet for Lie groups

Abstract: All semidirect products and functorial trivializations of first order and iterated bundles over a Lie group are presented. Symplectic reductions of iterated bundles by right invariance result in Tulczyjew's triplet for reduced manifolds. The trivialized Euler-Lagrange and Hamilton's equations are obtained and presented as Lagrangian submanifolds of the trivialized Tulczyjew's symplectic space. Euler-Poincaré and Lie-Poisson equations are presented as Lagrangian submanifolds of the reduced Tulczyjew's symplectic space. Tulczyjew's generalized Legendre transformations for trivialized and reduced dynamics are constructed.

Barbara Gołubowska, Wasyl Kowalczyk, Ewa Eliza Rożko
(Institute of Fundamental Technological Research Polish Academy of Sciences)
On affine motion and nonholonomic constraints

Abstract: In this work our goal is to carry out a thorough analysis of some geometric problems of the dynamics of affinely-rigid bodies. We present two ways to describe this case: the classical dynamical d'Alembert and variational (vakonomic) ones. So far, we can see that they give quite different results, but the vakonomic model from the mathematical point of view seems to be more elegant.

Jacek Jezierski (University of Warsaw)

(joined work with Piotr Waluk)

Proof of positive energy theorem by spacetime foliations

Abstract: Around 1961 R. Arnowitt, S. Deser, and W. Misner proposed, in their collaborative work, a way of defining “total four-momentum” of a gravitating system. The idea consisted in calculating a surface integral, constructed of metric derivatives, at infinity of some spatial hypersurface (“a slice of constant time”). This integral turns out to be well-defined and quite independent of deformations of the chosen hypersurface, as long as the choice is asymptotically flat, i.e., gravitation field falls off quickly enough at infinity.

$$g_{ab} = \delta_{ab} + h_{ab} \quad h_{ab} \in o(r^{-1})$$

The energy, or “mass”, component of ADM four-momentum turned out to be especially useful in various applications. It is given by a following integral:

$$M_{ADM} = \lim_{r \rightarrow \infty} \frac{1}{16\pi} \oint_{S(r)} (h^j_{k,j} - h^j_{j,k}) dS^k$$

In spite of importance of the concept, it took almost 20 years to settle such basic matter as the question of its positive definiteness.

It was only in 1979 that a complete proof was finally presented by Schoen and Yau, who succeeded by using variational arguments. Not much later, in 1981, another proof appeared (by Witten), based on the theory of spinors.

Here we present a yet alternative approach, requiring only basic tools of differential geometry.

Igor Kanatchikov (University of St Andrews)

From the polysymplectic structure to field quantization, YM mass gap and quantum gravity

Abstract: I outline the algebraic structures which can be obtained from the polysymplectic structure in field theory as generalizations of the Poisson bracket in mechanics. One of them is the Poisson-Gerstenhaber bracket of differential forms. I show how a quantization of a Heisenberg subalgebra of Poisson-Gerstenhaber algebra of forms leads to a construction of quantum fields viewed as sections of the Clifford bundle over the finite dimensional covariant configuration bundle of fields. I also outline how this reformulation of quantum field theory based on the mathematical structures of the De Donder-Weyl covariant Hamiltonian theory can be applied to the mass gap problem in quantum Yang-Mills theory and quantum gravity.

Antonio De Nicola (University of Coimbra)

Geometry and topology of cosymplectic spheres

Abstract: The notion of cosymplectic structure was introduced by P. Libermann in the late 50s as a pair (η, Ω) , where η is a closed 1-form and Ω a closed 2-form on an $2n + 1$ -dimensional manifold M , such that $\eta \wedge \Omega^n$ is a volume form. Cosymplectic manifolds play an important role in the geometric description of time-dependent mechanics (see [B. Cappelletti Montano, A. De Nicola, I. Yudin *Rev. Math. Phys.* **25** (2013), 1343002] and references therein). Starting from 1967, when Blair defined an adapted Riemannian structure on a cosymplectic manifold, a study of the metric properties on these manifolds was also initiated.

We study the geometry and topology of cosymplectic circles and cosymplectic spheres, which are the analogues in the cosymplectic setting of contact circles and contact spheres, introduced by [H. Geiges, J. Gonzalo *Invent. Math.* **121**, 147–209 (1995)], and then generalized by [M. Zessin *Ann. Inst.*

Fourier (Grenoble) **55**, 1167–1194 (2005)]. We provide a complete classification of 3-dimensional compact manifolds that admit a cosymplectic circle.

We introduce the notion of tautness and of roundness for a cosymplectic p -sphere. To any taut cosymplectic circle on a three-dimensional manifold M we are able to associate canonically a complex structure and a conformal symplectic couple on $M \times \mathbb{R}$.

In dimension three a cosymplectic circle is proved to be round if and only if it is taut. In higher dimensions we provide examples of cosymplectic circles which are taut but not round and examples of cosymplectic circles which are round but not taut. Finally we show that the three cosymplectic structures of any 3-cosymplectic manifold generate a cosymplectic sphere which is both round and taut.

Joana Nunes da Costa (University of Coimbra)
Triples of non-degenerate 2-forms on a Lie algebroid

Abstract: We show that starting with three non-degenerate 2-forms on a Lie algebroid that satisfy a simple condition, we may obtain several interesting structures.

Yunhe Sheng (School of Mathematics Jilin University)
Graded Poisson manifolds up to homotopy

Abstract: In this paper, we introduce a notion of a graded Poisson manifold up to homotopy, namely a Poisson $[n, k]$ -manifold, motivated by studying the dual of a Lie 2-algebra. We further study Maurer-Cartan elements on Poisson $[n, k]$ -manifolds and symplectic $[n, n]$ -manifolds. There are many interesting examples such as n -term L_∞ -algebras, twisted Poisson manifolds, quasi-Poisson \mathfrak{g} -manifolds and twisted Courant algebroids. As a byproduct, we justify that the symplectic $[n, n]$ -manifold is a homotopy version of the symplectic NQ-manifold and the Maurer-Cartan equation is a homotopy version of the master equation. The dual of an n -term L_∞ -algebra is a Poisson $[n, n]$ -manifold. We prove that the cotangent bundle of a Poisson $[n, n]$ -manifold is a symplectic NQ-manifold of degree $n + 1$. In particular, we construct a Courant algebroid from a 2-term L_∞ -algebra. By analyzing these structures, we obtain a Lie-quasi-Poisson groupoid from a Lie 2-algebra, which we propose to be the geometric structure on the dual of a Lie 2-algebra. At last, we obtain an Ikeda-Uchino algebroid from a 3-term L_∞ -algebra.

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(joint work with Y.-G. Oh (IBS Center for Geometry and Physics), H. V. Lê (Inst. of Math. at ASCR) and L. Vitagliano (University of Salerno))
Deformations of coisotropic submanifolds in abstract Jacobi manifolds

Abstract: In this work, using the Atiyah algebroid and first order multi-differential calculus on non-trivial line bundles, we attach an L_∞ -algebra to any coisotropic submanifold S in an abstract (or Kirillov's) Jacobi manifold. Our construction generalizes and unifies analogous constructions in the symplectic case (Oh and Park), the Poisson case (Cattaneo and Felder), locally conformal symplectic case (Lê and Oh). As a new special case, we attach an L_∞ -algebra to any coisotropic submanifold in a contact manifold, including Legendrian submanifolds. The L_∞ -algebra of a coisotropic submanifold S governs the (formal) deformation problem of S .

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