DIFCOCA and Secondary Calculus

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PART 1:
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ORDERING SOME COMMON PLACES AND BANALITIES

What is the meaning to (scientific) life ?
What we have to do?
Where and how to begin?

Common places (axioms) no.1 and no.2

Any exact knowledge is mathematics but not vice versa

$$\Downarrow$$

Any explanation presupposes a language and by this reason mathematics is a system of languages that were historically formed in the course of various attempts to understand Nature.

$$\Downarrow$$

Formation of an adequate mathematical language is a long and tortuous process, which is governed by Darwin's selection mechanism. This is mainly due to the natural ignorance of Wittgenstein's principle.

Wittgenstein principle

"Limits of my language are limits of my world."

$$\Downarrow$$

Whether a new (physical) reality can be adequately described in terms of already existing mathematics

Wittgenstein horizon

Wittgenstein horizon is the boundary of what can be adequately described by a given language

$$\Downarrow$$

What is beyond the Wittgenstein horizon are fantasies even if written in terms of (very sophisticated) mathematical formulas

ACTUAL QUESTION:

Which physics/mathematics is beyond the Wittgenstein horizon of the contemporary mathematics?

Examples

1. Zeno paradoxes $\Rightarrow$ Differential Calculus
2. Trisecting the general angle by a ruler and compass construction (from the history of USSR)
3. Turbulence: failure of analytical approaches
4. J.von Neumann vs generalized Bohr's correspondence principle
5. Quantum gravity and gravitons, dark matter and dark energy, gravitational waves,...
6. Deformation quantization
7. Non-commutative geometry and physics
8. Sheaves and complex manifolds
9. Differential algebraic geometry
10. etc ...

$\Downarrow$

We are living in the epoch of
TERMINOLOGICAL PHYSICS ?!

Information chaos and what to do?

In the situation of chaotic and uncontrolled production of mathematical facts and theories, great parts of which sooner or later disappear from the circulation, is it possible to see an order and structure?

$$\Downarrow$$

What humans could do in the situation
when not humans but the Darwin-like selection mechanism is
who takes the decision?

$$\Downarrow$$

In particular, is it possible to pass from the natural selection to more efficient artificial one?

Initial data and where to start?

AMS subject classification (AMS-SC) is an attempt to structure contemporary mathematics.
(Mind also the idea to put set theory into foundations of all mathematics, Hilbert's paradise and N.Bourbaki)

$\Downarrow$

AMS-SC is something similar to Carl Linnaeus taxonomy (CLT) in biology but a comparison is not in favor of it.

$\Downarrow$

This is my critics of AMS-SC :

....

....

....!!!

$\Downarrow$

One of the reasons is that progress in biology have led to genetics,
the general theory of living things,
which gives the basis to understand their diversity, properties, etc,
in sharp contrast with what is happening in mathematics.

In the area of our interest we see a zoo of geometrical structures and (N)PDEs,
which are mainly studied as single “animals" of some practical interest

$\Downarrow$

MAIN QUESTION

Is it possible to develop "genetics" for geometrical structures and to build on this basis a pithy general theory of NPDEs ?

PART 2.

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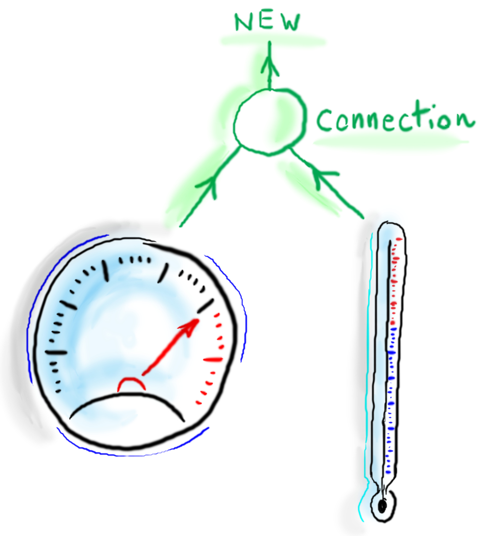
From observability in classical physics to Di COCA

(In search of "Universal Language")

OBSERVATIONS $\Rightarrow$ LABORATORY



Classical laboratory is a commutative algebra

Measuring devices generate an unitary commutative algebra A over $\mathbb{R}$:

- $+$
- $\cdot$
- zero.

OBSERVATION is a homomorphism $f : A \longrightarrow \mathbb{R}$ of unitary $\mathbb{R}$–algebras.

STATES SPACE OF THE PHYSICAL SYSTEM IN QUESTION

$$\mathrm{Spec}_{\mathbb{R}} A \stackrel{\text{def}}{=} \{\text{all } h : A \longrightarrow \mathbb{R}\}$$

Any element $a \in A$ is a function on $\mathrm{Spec}_{\mathbb{R}} A$:

$$a(h) \stackrel{\text{def}}{=} h(A)!$$

Theorem

$$A = C^{\infty}(M) \Rightarrow \boxed{\mathrm{Spec}_{\mathbb{R}} A = M}.$$

$$M \ni x \longmapsto h_x \in \mathrm{Spec}_{\mathbb{R}} A,$$

$h_x(f) = f(x)$, $f \in A = C^{\infty}(M)$.
(not $f(x)$, but $x(f)$!)

Zarissky topology on $\mathrm{Spec}_{\mathbb{R}} A$

$$A \ni a \longmapsto \underset{\text{open}}{U_a} \subseteq \mathrm{Spec}_{R} A, \quad U_a \stackrel{\text{def}}{=} \{h \in \mathrm{Spec}_{R} A \mid h(a) \neq 0\}.$$

Example

$$A = C^{\infty}(M), \quad a = f, \quad , U_f = \{h_x \in \mathrm{Spec}_{R}(A) = M \mid f(x) \neq 0\}.$$

Theorem

Zarissky topology on $\mathrm{Spec}_{\mathbb{R}} C^{\infty}(M) = M$ coincides with the standard one.

All above is valid for any unitary algebra A over an algebraic field $\mathbb{K}$:

$$\mathrm{Spec}_{\mathbb{K}} A = \{\text{all } h : A \longrightarrow \mathbb{K}\}, \quad \text{etc.}$$

FUNDAMENTAL TEST

Since differential calculus is a unique natural language for classical physics, it must be an aspect of commutative algebra if the above observation mechanism is true.

$\Downarrow$ YES!

$A =$ an unitary $\mathbb{K}$–algebra, P, Q are A–modules.

MAIN DEFINITION

$\Delta : P \longrightarrow Q$ is a linear D.O. of order $\leq m$ if Δ is $\mathbb{K}$–linear and $[a_0, [a_1, \dots, [a_m, \Delta] \dots]] = 0$, $\forall a_0, a_1, \dots, a_m \in A$.

MAIN THEOREM

If $A = C^\infty(M)$, $\mathbb{K} = \mathbb{R}$, $P = \Gamma(\pi)$, $Q = \Gamma(\pi')$, π, π' being vector bundles over M, then MAIN DEFINITION gives usual D.O.'s.

$\Downarrow$ IMMEDIATE GENERALIZATIONS

GRADED (in particular SUPER), FILTERED, . . . , COMMUTATIVE ALGEBRAS.

EXAMLPE

Smooth sets: $N \subseteq M$ (manifold) – a closed subset.

$$A = C^\infty(N) \overset{\text{def}}{=} \{f|_N \mid f \in C^\infty(M)\}.$$

One can develop differential calculus over Kantor's set, Peano curve, etc.

THEOREM

Differential operators over a Boolean algebra are of order 0.

$$\Downarrow$$

Phenomena of motion, evolution, etc., cannot be expressed in terms of usual/natural languages (LOGIC).

$$\Downarrow$$

Before Newton–Leibniz mechanics ($\cong$ physics) was beyond the Wittgenstein horizon of the epoch.

$$\Downarrow$$

Stone spaces, observable sets, paradoxes of set theory and Hilbert's paradise

“LOGIC” OF DIFFERENTIAL CALCULUS OVER COMMUTATIVE ALGEBRAS

A = unitary $\mathbb{K}$–algebra, P, Q – A–modules.
Important particular case $\mathrm{A} = \mathrm{C}^\infty(\mathrm{M})$, $\mathbb{K} = \mathbb{R}$, $\mathrm{P} = \Gamma(\pi)$, $\mathrm{Q} = \Gamma(\pi')$.

Basic notation

$$\mathit{Diff}_k(P,Q) \stackrel{def}{=} \{\text{all D.O.'s of order} \leq \text{k from P to Q}\}$$

$$\mathit{Hom}_A(P,Q) = \mathit{Diff}_0(P,Q)$$

$$\mathit{Diff}_0(P,Q) \subset \mathit{Diff}_1(P,Q) \subset \cdots \subset \mathit{Diff}_k(P,Q) \subset \cdots \subset \underbrace{\mathit{Diff}(P,Q)}_{\text{ALL}}$$

$$\mathit{Diff}_k^<(P,Q) - \text{left } A\text{–module structure}$$

$$\mathit{Diff}_k^>(P,Q) - \text{right } A\text{–module structure}$$

$$\mathit{Diff}_k^<(P) \stackrel{def}{=} \mathit{Diff}_k^<(A,P), \quad \mathit{Diff}_k^>(P) \stackrel{def}{=} \mathit{Diff}_k^>(A,P)$$

$\mathit{Diff}_k^<(P,Q) \xrightarrow{id} \mathit{Diff}_k^>(P,Q)$ *is* DO of order k!

Derivations: the simplest functors of differential calculus

$$D(P) = \{\Delta \in Diff_1^{<}(P) \mid \Delta(1) = 0\} \equiv$$

$$\equiv \{\Delta : A \longrightarrow P \mid \underbrace{\Delta(ab) = a\Delta(b) + b\Delta(a)}_{\text{derivations}}\}$$

$D(A)$ are vector fields on $Spec\, A$.
If $A = C^\infty(M)$, then $D(A) = \{\text{vector fields on } M\}$.

$$D_2(P) \stackrel{def}{=} D^{<}(D(P) \subset Diff_1^{>} P), \quad \mathcal{P}_2^{<>}(P) = Diff_1^{<>}(D(P) \subset Diff_1^{>} P)$$

If $A = C^\infty(M)$, then $D_2(A)$ are bivector fields on M.

$$D_m(P) \stackrel{def}{=} D^{<}(D_{m-1}(P) \subset \mathcal{P}_{m-1}^{>} P),$$

$$\mathcal{P}_m^{<>}(P) = Diff_1^{<>}(D_{m-1}(P) \subset \mathcal{P}_{m-1}^{>} P)$$

$A = C^\infty(M) \Rightarrow D_m(A) = \{m - \text{vector fields on } M\}$.

FUNCTORS OF DIFFERENTIAL CALCULUS (FUDICs)

$$D : P \longmapsto D(P), \quad Diff_k^{>} : P \longmapsto Diff_k^{>}(P)$$

$$Diff_k^{<}(\cdot,\cdot) \colon P, Q \mapsto Diff_k^{<}(P, Q), \quad \text{ETC...}$$

HIGHER ANALOGUES OF MULTIVECTORS

$$D_{(k)}(P) \overset{def}{=} \{\Delta \in Diff_k^{<} P \mid \Delta(1) = 0\}, \quad \mathcal{P}_{(k)}^{<>}(P) \overset{def}{=} Diff_k^{<>}(P)$$

$$D_{(k,l)}(P) \overset{def}{=} D_{(k)}^{<>}(D_{(l)}(P) \subset Diff_l^{>}(P))$$

$$\mathcal{P}_{(k,l)}^{<>}(P) \overset{def}{=} Diff_k^{<>}(D_{(l)}(P) \subset Diff_l^{>}(P))$$

$$\Downarrow$$

$$\text{inductively} \quad D_{(k,l,\dots,m)}, \quad \mathcal{P}_{(k,l,\dots,m)}^{<>}$$

These functors are beyond Wittgenstein's horizon of the ordinary Dif. Geometry

HOMOMORPHISMS OF FUNCTORS: EXAMPLES

$$D \hookrightarrow Diff_1^<, \quad D_2 \hookrightarrow D^<(Diff_1^>) \hookrightarrow Diff_1^<(Diff_1^>)\ldots$$

$$Diff_k^{<>} \hookrightarrow Diff_l^{<>}, \quad l \geqslant k$$

$$Diff_k^<(Diff_l^>) \longrightarrow Diff_{k+l}^<, \; D_{k+l} \hookrightarrow D_k(D_l)$$

$$D_{k+1} \to D_k^<(Diff_1^>) \quad \Rightarrow \quad D_{k+1}^<(Diff_{l-1}^>) \to D_k^<(Diff_l^>)$$

DIFFERENTIAL OPERATORS ON FUNCTORS

If Φ, Ψ are some fundics, then

A DO from Φ to Ψ is a homomorphism $\quad \Phi \longrightarrow \Psi^<(Diff_k^>)$

Do you see something non-banal in the following tautology?

$$Diff_k^< \xrightarrow{id} id^<(Diff_k^>) = Diff_k^<$$

THE ABOVE IS THE ALGEBRA OF ABSOLUTE FUNCTORS
OF DIFFERENTIAL CALCULUS

THE "LOGIC" OF DIFFERENTIAL CALCULUS
CONSISTS OF FUNDICS AND THEIR DO'S

NOTE THAT IT DOES NOT DEPENDS ON CONCRETE
COMMUTATIVE ALGEBRAS !!!

ALL STRUCTURES IN CLASSICAL DIFFERENTIAL GEOMETRY
ARE
CONSTRUCTIONS IN THIS "LOGIC"

ALL FUNDAMENTAL EQUATIONS IN NATURAL SCIENCES
ARE
ASSERTIONS IN TERMS OF THIS "LOGIC"

.

In order to study a concrete situation this "logic algebra" should be
represented/"materialized"
in a suitable category of modules $\mathcal{K}$
over a suitable commutative algebra A (of "observables").

$$\Downarrow$$

This, in particular, include representation of fundics in $\mathcal{K}$

$$\Downarrow$$

$\mathcal{O}_\Phi \in Ob(\mathcal{K})$ represents a fundic Φ if

$$\Phi(A) = Hom_A(\mathcal{O}_\Phi, P), \quad \forall P \in Ob(\mathcal{K})$$

Elements of representing objects are "covariant" quantities,
while that of fundics are "contravariant" ones

Examples

1-forms

Functor D: $\exists!$ the universal derivation $d = d(A) : A \to \Lambda^1(A)$.

$\boxed{X \in D(P)}$

$$\begin{array}{ccc} A & \xrightarrow{d} & \Lambda^1(A) \\ {\scriptstyle X}\downarrow & \swarrow{\scriptstyle h_X} & \\ P & & \end{array}$$

$\boxed{h_X \text{ – homomorphism of } A\text{–modules}}$

$\Lambda^1(A)$ represents D in $\mathcal{M}(A)$.

k-jets

Functor $Diff_k^<$: universal k–th order DO $j_k = j_k(A) : A \to \mathcal{J}^k(A)$.

$\boxed{\Delta \in Diff_k(P)}$

$$\begin{array}{ccc} A & \xrightarrow{j_k} & \mathcal{J}^k(A) \\ {\scriptstyle \Delta}\downarrow & \swarrow{\scriptstyle h_\Delta} & \\ P & & \end{array}$$

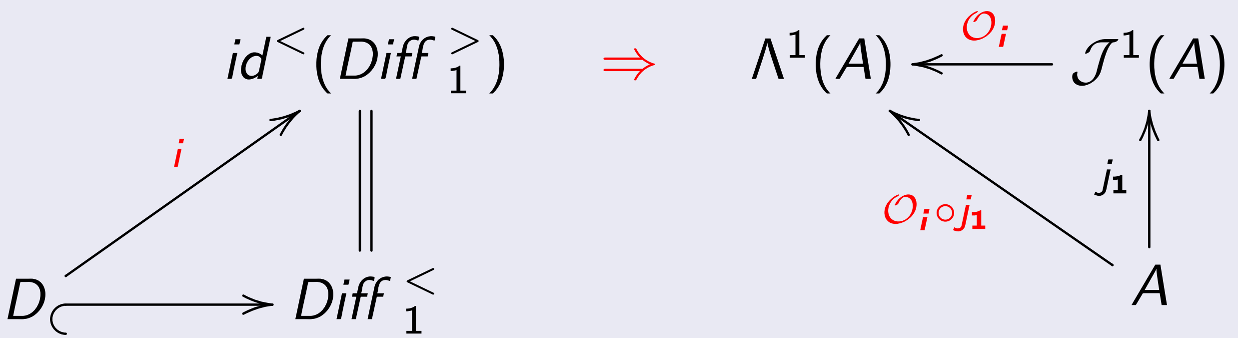
$\boxed{h_\Delta \text{ – homomorphism of } A\text{–modules}}$

$$\underset{\text{a map of functors}}{\Phi \xrightarrow{\alpha} \Psi} \quad \overset{\text{induces}}{\Longrightarrow} \quad \underset{\text{a homomorphism of representing objects}}{\mathcal{O}_\Phi \xleftarrow{\mathcal{O}_\alpha} \mathcal{O}_\Psi}$$

$$\Phi \xrightarrow{\alpha} \Psi(Diff_{\,k}^{\,>}) \Rightarrow \begin{array}{ccc} \mathcal{O}_\Phi & \xleftarrow{\quad\mathcal{O}_\alpha\quad} & \mathcal{J}^k(\mathcal{O}_\Psi) \\ & \nwarrow^{\mathcal{O}_\alpha \circ j_k} & \uparrow j_k \\ & & \mathcal{O}_\Psi \end{array}$$

$\mathcal{O}_\alpha \circ j_k$ – natural k–th order DO.

EXAMPLE



If $A = C\infty(M)$, then $\mathcal{O}_i \circ j_1 = d : C^\infty(M) \to \Lambda^1(M)$.

EXAMPLE

$$D_2 \overset{\nu}{\hookrightarrow} D^{<}(\mathit{Diff}\,{}^{>}_{1}) \quad \Rightarrow \quad \begin{array}{ccc} \Lambda^2(A) & \xleftarrow{\;\mathcal{O}_\nu\;} & \mathcal{J}^1(\Lambda^1) \\ & {\scriptstyle \mathcal{O}_\nu \circ j_1} \nwarrow & \uparrow {\scriptstyle j_1} \\ & & \Lambda^1 = \Lambda^1(A) \end{array}$$

If $A = C\infty(M)$, then $\mathcal{O}_\nu \circ j_1$ is
the exterior differential $d : \Lambda^1(M) \to \Lambda^2(M)$.

etc...

All that can be done for smooth sets, supermanifolds, etc., etc....

REPRESENTING OBJECTS DEPEND ON CATEGORY OF MODULES

$$A = C^\infty(M)$$

$\Lambda^1_{alg}(A)$ represents D in the category of all A-modules,
$\Lambda^1_{geom}(A)$ represents D in the category of geometric A-modules

$$\Downarrow$$

There is a natural surjection $\Lambda^1_{alg}(A) \longrightarrow \Lambda^1_{geom}(A)$ whose kernel is highly nontrivial. For instance,

$$d_{alg} f - f' d_{alg} \neq 0 \text{ if } x \text{ and } f(x) \text{ are algebraically independent}$$

and

$$\Lambda^1_{geom}(A) = \Lambda^1(M) = \{\text{standard 1-forms}\}$$

If $\mathcal{K}$ is the category of $C^\infty(M)$-modules with one point support at $x \in M$, then

$$d_{\mathcal{K}} = 0 \text{ and } \Lambda^1_{\mathcal{K}}(A) = T^*_x(M)$$

$$\Downarrow$$

A.Grothendick bloks differential algebraic geometry

$$\Downarrow$$

FUNDAMENTAL PROBLEM: To transform ALGEBRAIC GEOMETRY
into
DIFFERENTIAL ALGEBRAIC GEOMETRY

Digression: DOWN WITH SHEAVES !

The spectrum of the algebra of holomorphic functions $Hol(M)$ on a complex manifold M is rather poor and, generally, is different from M.
$\Rightarrow$ Opens on M can not be observed by means of instruments in the "laboratory" $Hol(M)$ as well as all holomorphic objects.
$\Rightarrow$ Complex manifolds are defines by sewing together euclidean pieces ("patchwork quilt" !). Sheaves appeared as a way to define various holomorphic cohomologies on such a quilt.

$$\Uparrow$$

If a complex manifold is defined as $C^\infty(M)$ supplied with
an integrable Nijenhuis tensor N,

then all this staff is easily obtained as objects that are compatible with N.

$$\Downarrow$$

Toward complex analysis and geometry over arbitrary commutative algebras !

HAMILTONIAN MECHANICS OVER COMMUTATIVE ALGEBRAS

- $Spec_{\,k} A$="configuration space" $\Leftarrow$ With A "positions" of the mechanical system
- $A = Diff_{\,0} A \subset Diff_{\,1} A \subset \ldots Diff_{\,k} A \subset \ldots Diff\, A$= $\Leftarrow$ filtered algebra
- $smbl_{\,k}\Delta = \frac{Diff_{\,k} A}{Diff_{\,k-1} A} \Rightarrow smbl\, A = \bigoplus_{k \geqslant 0} smbl_{\,k} A$ $\Leftarrow$ associated graded algebra
- $\Delta \in Diff_{\,k} A \Rightarrow smbl_{\,k}\Delta \in smbl_{\,k} A$= $\Leftarrow$ principal symbol of Δ
- $smbl_{\,k}\Delta \cdot smbl_{\,l}\Box = smbl_{\,k+l}\Delta \circ \Box$= $\Leftarrow$ multipliction in $Smbl\, A$

FACT

$$\Delta \in Diff_k A, \quad \square \in Diff_l A \Rightarrow [\Delta, \square] \in Diff_{k+l-1} A$$
$$\Downarrow$$
$$smbl\, A \text{ is commutative}$$

THEOREM

If $A = C^\infty(M)$, then $Spec_{\mathbb{R}}(Smbl\, A) = T^*M$

$$\{smbl_k \Delta, smbl_l \square\} \stackrel{def}{=} smbl_{k+l-1}([\Delta, \square]) \Leftarrow$$ Poisson bracket on $T^*(Spec_k A)$

This is the standard Poisson bracket on T^*M if $A = C^\infty(M)$.

- configuration space $= Spec_k A$
- phase space (coordinates + momenta) $= Spec_k(Smbl\, A)$
- kinetic energy $= smbl_2 \Box$ ("metric" on $Spec_k A$)
- potential energy $= smbl_0 \Delta \in Smbl_0 A = A$
- Hamiltonian $= smbl_s \Box + smbl_0 \Delta = H \Rightarrow X_H = \{H, \cdot\}$
  Hamiltonian field

Immediate construction of RATIONAL MECHANICS on cross, “eight”, Peano curve, super–manifolds, etc.
It is easy to discover new laws of rational mechanics (collisions and other impulsive motions)

$\Downarrow$ NO NEED OF “FORCES”, LEGENDRE TRANSFORM, ...

one step to orbit method is Lie algebra representation, “geometric quantization”, ...

WHAT IS THE ROLE OF SYMBOLS OF PDE's?

- $\{smbl_k\Delta, \cdot\} = X_{smbl\,_k\Delta}$ –Hamiltonian field
- $smbl_k\Delta = 0$ –corresponding Hamilton-Jacobi equation
- $A \ni a$ is a solution of this equation iff $\underbrace{[a, \cdots [a, [a, \Delta]] \cdots]}_{k-times} = 0$

Trajectories of $X_{smbl\,_k\Delta}$ are characteristics of $smbl\,_k\Delta = 0$

INTERPRETATION

Solutions of $smbl\,_k\Delta = 0$ are "wave fronts" of solutions of $\Delta = 0$ which propagate along characteristics of $X_{smbl\,_k\Delta}$.

AND

WAVE FRONTS ARE SHADOWS IN SPACE-TIME OF MULTI-VALUES SOLUTIONS SINGULARITIES OF $\Delta = 0$

CRUCIAL POINT

$\Downarrow$

TERRA INCOGNITA

WHAT ARE THESE SINGULARITIES, THEIR KINDS, PHYSICAL MEANING,...

EXAMPLE

Fold–type singularities for $u_{xx} - \frac{1}{c^2} u_{tt} - mu^2 = 0$.

Wave fronts in the form $x = \varphi(t)$

$$y = u|_{\text{wavefront}}, \quad h = u_x|_{\text{wave front}}$$

$$\Downarrow$$

$$\begin{cases} \ddot{y} + (cm)^2 g = \pm 2c\dot{h} \\ 1 - \frac{1}{c^2}\dot{\varphi}^2 = 0 \Leftrightarrow \dot{\varphi} = \pm c \end{cases} \Leftarrow \left[\text{Equations describing behaviour of fold–type singularities} \right.$$

In particular: $\dot{h} = 0$ (resting "particle") $\Rightarrow \ddot{g} + (cm)^2 g = 0 \Rightarrow \boxed{\nu = mc}$

SOME HISTORY

Maxwell $\to$ Luneburg
Schrödinger $\to$ Levi-Civita
$\Downarrow$
THEORY OF SINGULARITIES OF SOLUTIONS

$\mathcal{E}$=PDE, Σ = singularity type
$\Downarrow$
$\mathcal{E}_\Sigma$ = eq. describing shapes of singularities of solutions of $\mathcal{E}$.
WAVEFRONTS CORRESPOND TO Σ=FOLD.

EXAMPLE

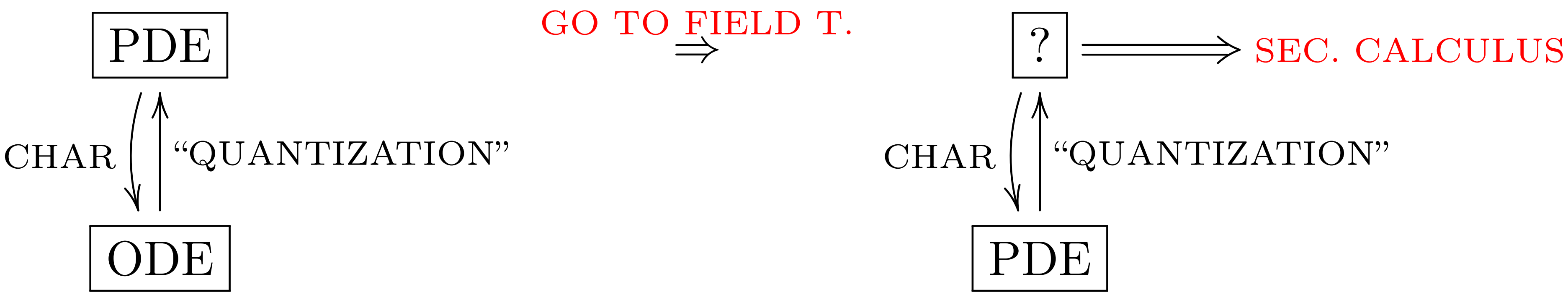
$\mathcal{E}$ = Klein–Gordon: $(\partial_t^2 - \vec{\nabla}^2 + m^2)u = 0$

wave front in the form $\{t = \varphi(x_1, x_2, x_3)\} = S$

$$\mathcal{E}_{\mathrm{FOLD}} = \begin{cases} (\vec{\nabla}\varphi)^2 = 1 & \text{eiconal type eq.} \\ \nabla^2 h + m^2 h - g - (\nabla^2\varphi)g = 2\vec{\nabla}\varphi \cdot \vec{\nabla}g \leftarrow ???? \end{cases}$$

THEOREM

For hyperbolic $\mathcal{E}$: $\mathcal{E}_\Sigma$ characterizes completely $\mathcal{E}$.



$$\mathrm{CHAR} : \mathcal{E} \mapsto \mathcal{E}_{\mathrm{FOLD}} \mapsto \mathrm{Eiconal}.$$

WHAT IS A RIEMMANNIAN METRIC?

LEVI-CIVITA CONNECTION PROBLEM

$A \Rightarrow \overbrace{\Lambda_{(1)} = \Lambda(A)}^{\text{DIFF. FORMS OVER } A} \Rightarrow \overbrace{\Lambda_{(2)} = \Lambda(\Lambda_{(1)})}^{\ldots\text{OVER } \Lambda_{(1)}\ldots} \Rightarrow \cdots \Rightarrow \Lambda(\Lambda_{(k-1)}) \Rightarrow \cdots \Lambda_{(\infty)}$

THEOREM

$\Lambda(\Lambda_{(\infty)}) \cong \Lambda_{(\infty)}$.

"*proof*": $\infty + 1 = \infty$. □

$$\{\Lambda_{(1)}, d_1 = d\},\ \{\Lambda_{(2)}, d_2, d_1\},\ \ldots,\ \{\Lambda_{(k)}, d_k, \ldots, d_1\},\ \ldots$$
$$d_1 \leftrightarrow L_{d_1}.$$

Example

Tensors: $t_{i_1\dots i_k}dx^{i_1}\otimes\cdots\otimes dx^{i_k} \Rightarrow \overbrace{\underbrace{t_{i_1\dots i_k}d_1x^{i_1}}_{\text{in }\Lambda_{(k-1)}}\cdots d_kx^{i_k}}^{\text{in }\Lambda^1_{(k)}}$

Example

$\Lambda^1_{(2)}$: $\overbrace{\omega_{i_1\dots i_k}dx^{i_1}\wedge\cdots\wedge dx^{i_k}}^{\text{“FUNCTIONS”}\in\Lambda_{(1)}}$, $\qquad$ DIFFERENTIALS $d_2d_1x^i$, $\quad d_2x^j$

RIEMANNIAN METRIC $g_{ij}dx^i\otimes dx^j \Rightarrow \boxed{g = g_{ij}d_1x^i\cdot d_2x^j}$!

LEVI-CIVITA CONNECTION

$$\Gamma = (\text{“}g^{-1}\text{”} \circ d_2 \circ d_1)(g) = (d_1 d_2 x^\alpha + \Gamma^\alpha_{\beta\gamma} d_1 x^\beta \cdot d_2 x^\gamma) i_{\partial_\alpha}$$

IN $D(\Lambda_{(1)}, \Lambda_{(2)})$

COMPARE WITH $\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma} \dot{x}^\beta \dot{x}^\gamma$ (GEODESIC CURVATURE)

$$R = [\Gamma, \Gamma]^{\mathrm{FN}}, \ldots$$

Usual vacuum Einstein eq. $\Rightarrow \mathrm{Ric}(g) = 0$

Introduce a kind of matter by passing from g to T_0^2

$$\Downarrow$$

$$T_0^2 \ni t = \overset{\text{sym}}{g} + \underset{\text{matter}}{\overset{\text{skew}}{\omega}}$$

$$\boxed{\mathrm{Ric}(t) = 0} \Rightarrow \begin{cases} \mathrm{Ric}(g) + \frac{g}{16} g^{ij} g^{kl} \partial_{[} \omega_{pj]} \partial_{[i} \omega_{ql]} = 0 \\ \underset{\text{perfect fluid}}{\overset{g}{\nabla}{}^{i}} (\partial_{[j} \omega_{ki]}) = 0 \end{cases}$$

$$\Uparrow$$

DARK ENERGY (?)

WHAT TO DO?

- Differential geometry on simplicial complexes
- Graded algebras
- Machanics on graded algebras