# Mathematics behind the Nobel Prize in Physics 2020

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# Official announcement

The **Nobel Prize in Physics** 2020 was divided,

- one half awarded to **Roger Penrose** "for the discovery that black hole formation is a robust prediction of the general theory of relativity",
- the other half jointly to **Reinhard Genzel** and **Andrea Ghez** "for the discovery of a supermassive compact object at the centre of our galaxy".

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Sagittarius A* (**SgrA***) is a very bright and **very compact** (less than 0.1 arc sec) **radio source in the center of the Milky Way**; it is known since 1974.

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- Reinhard Genzel and his Munich group, were the first to estimate the *mass* of this object, to be $M = 4.31 \pm 0.06 \times 10^6\, M_\odot$; because of its *compactness* SgrA* is believed to be a supermassive *black hole*;
- Genzel's group result was based on observations of a star $S2$, which is in the distance smaller that 17 *light hours* from the SgrA*; in the years 1992-2002 they observed in infrared $2/3$ of the full revolution of $S2$ around SgrA*; from this they reconstructed a *Keplerian orbit* of $S2$; then they used the **third Kepler's law**:
  *the square $T^2$ of star's orbital period $T$ is proportional to the cube $a^3$ of the length $a$ of star's orbit semi-major axis*; actually, modulo an universal constant $c$, we have $\frac{a^3}{T^2} = \mathrm{c}M$, so knowing $a$ and $T$ we know the gravitational source mass $M$;

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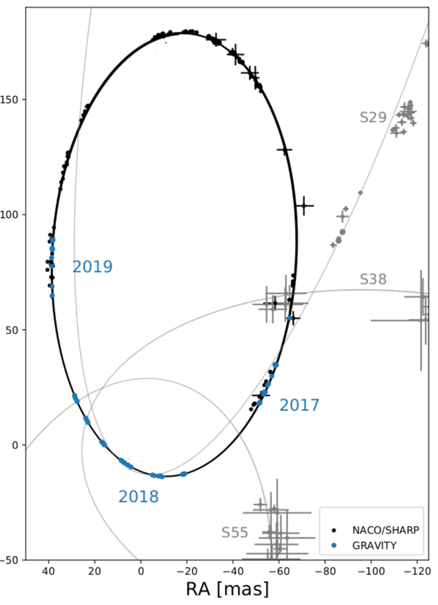
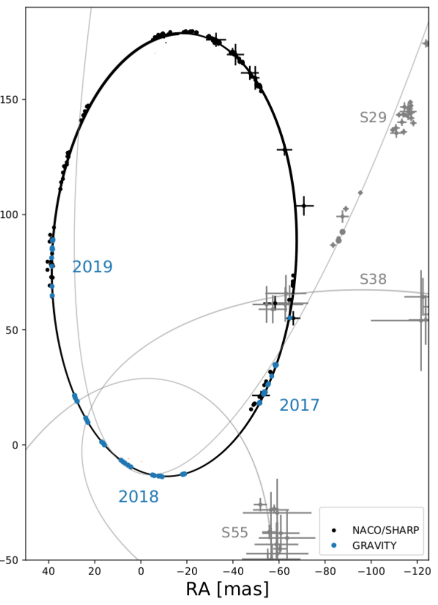



- The Keplerian orbit of the *S2* star in the vicinity of SgrA*.
- The blue orbit points are measured by GRAVITY - a second - generation instrument of the Very Large Telescope Interferometer (VLTI) at ESO's Paranal Observatory, Chile, operating since 2016. As one of ESO astronomers says: 'It is born from the desire to observe very small details of faint objects, including those at the centre of galaxies. With its high sensitivity and accuracy, GRAVITY can reveal a whole new world of planets, stars and galactic centres that were previously out of reach because they were too faint for previous instruments'.
- The picture is prepared by an astronomer Odele Straub, a member of Reinhard Genzel's group since 2017.
- Odele made this picture for us, especially for this talk.

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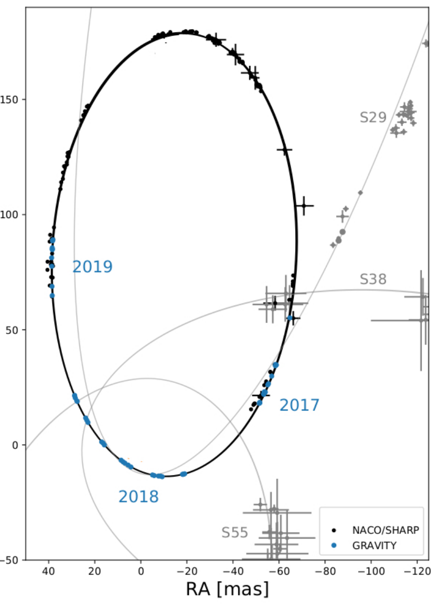


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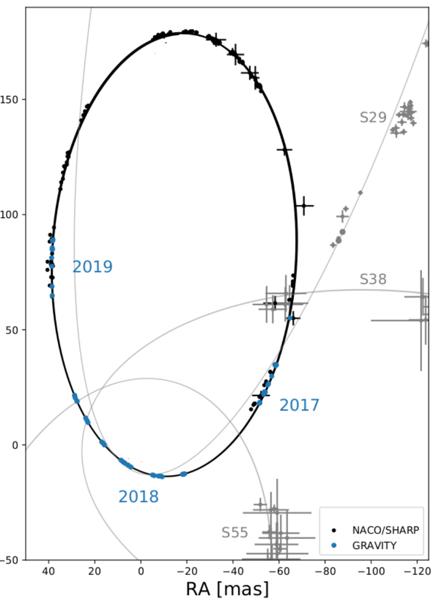


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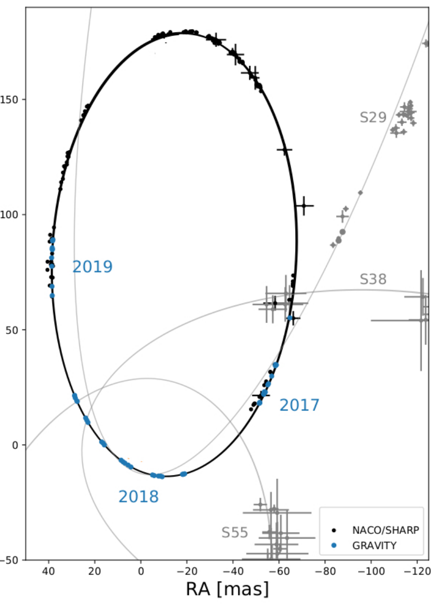


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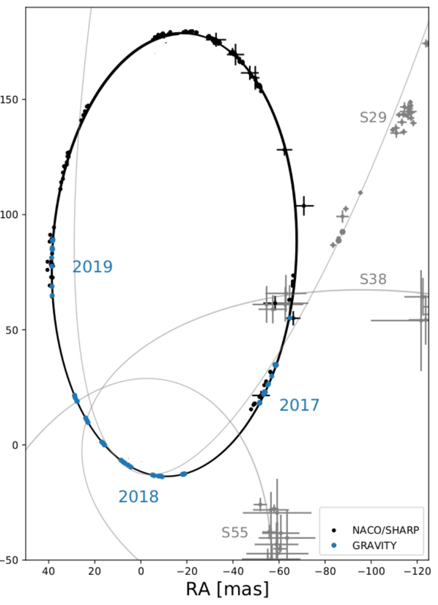


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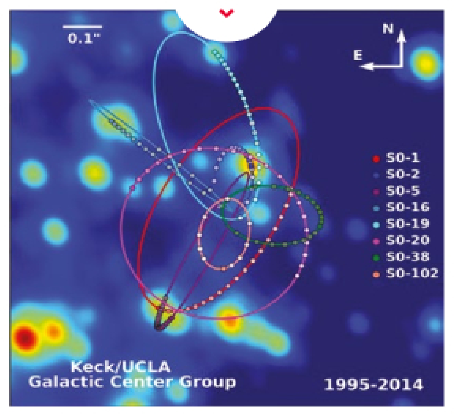


- Keplerian orbits of eight stars close to SgrA*
- The infrared photograps of these stars where obtained by the group of Andrea Ghez during 20 years, using the Keck telescope;
- The Keck telescope consists of two twin 10-meter in diameter telescopes; it is one of the most precise and most technologically adavanced telescope on our planet. It is situated at mount Mauna Kea, Hawaii, at the altitude of 4100m
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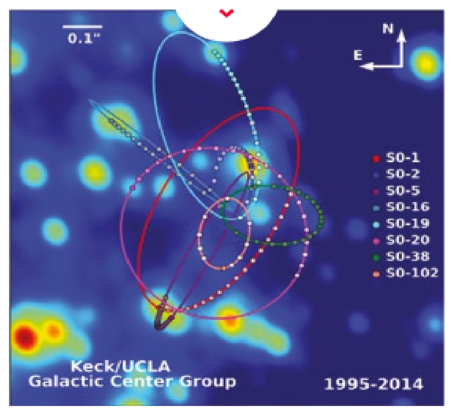


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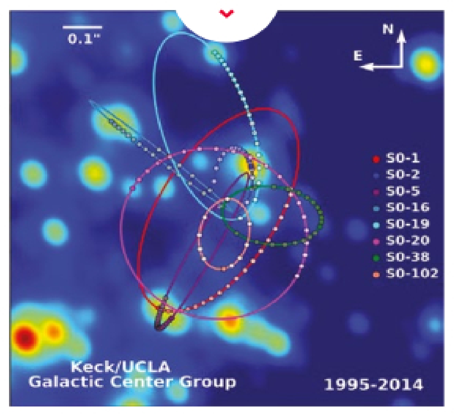


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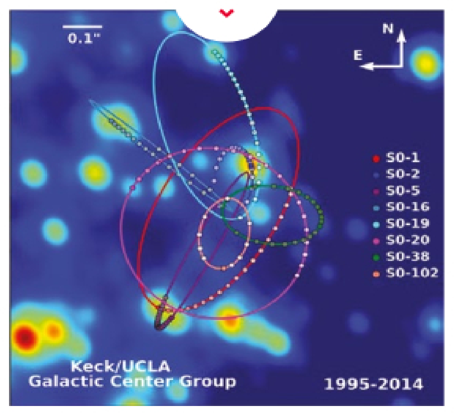


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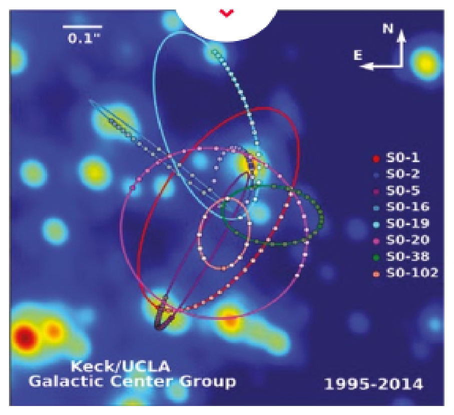


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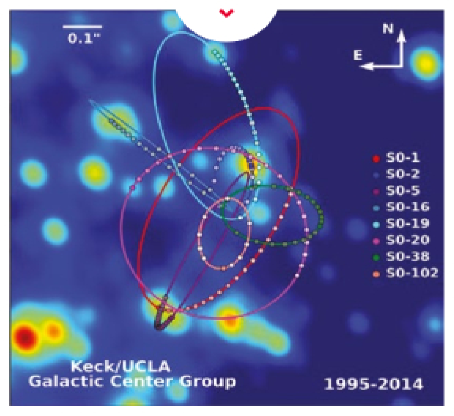


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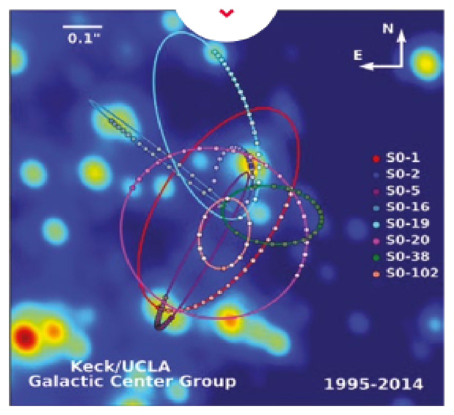


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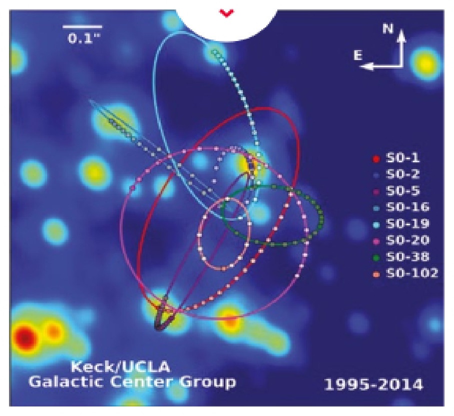


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# Comments from Odele Straub

- Ghez and Genzel are both infrared astronomers, leading teams of observers, theoreticians and engineers. However, one would perhaps like to add the radio observer Mark Reid to the laureates, as he could show that the BH is supermassive by measuring how it moves (it nearly doesn't).
- The maths behind this discovery has many faces. Typically we use both, PPN (O(1)) and full GR.
- The old pre-GRAVITY data for the star S2 have too large uncertainties to access the BH spin today. We have measured one full orbit, but it has period of 16 years. We think that we can access the spin in principle after a few full orbits taking into account the accumulative nature of the Lense-Thirring effect on the S2 star's trajectory. However, there will also be Newtonian effects of the neighbouring S-stars of that order of magnitude, which have to be considered - and they work in the opposite direction to the LT effect. The full problem is highly non-trivial.

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# Nobel Prize paper

VOLUME 14, NUMBER 3 PHYSICAL REVIEW LETTERS 18 JANUARY 1965

## GRAVITATIONAL COLLAPSE AND SPACE-TIME SINGULARITIES

Roger Penrose
Department of Mathematics, Birkbeck College, London, England
(Received 18 December 1964)

The discovery of the quasistellar radio sources has stimulated renewed interest in the question of gravitational collapse. It has been suggested by some authors[1] that the enormous amounts of energy that these objects apparently emit may result from the collapse of a mass of the order of $(10^6\text{-}10^8)M_\odot$ to the neighborhood of its Schwarzschild radius, accompanied by a violent release of energy, possibly in the form of gravitational radiation. The detailed mathematical discussion of such situations is difficult since the full complexity of general relativity is required. Consequently, most exact calculations concerned with the implications of gravitational collapse have employed the simplifying assumption of spherical symmetry. Unfortunately, this precludes any detailed discussion of gravitational radiation—which requires at least a quadripole structure.

The general situation with regard to a spherically symmetrical body is well known.[2] For a sufficiently great mass, there is no final equilibrium state. When sufficient thermal energy has been radiated away, the body contracts and continues to contract until a physical singularity is encountered at $r=0$. As measured by local comoving observers, the body passes within its Schwarzschild radius $r=2m$. (The densities at which this happens need not be enormously high if the total mass is large enough.) To an outside observer the contraction to $r=2m$ appears to take an infinite time. Nevertheless, the existence of a singularity presents a serious problem for any complete discussion of the physics of the interior region.

The question has been raised as to whether this singularity is, in fact, simply a property of the high symmetry assumed. The matter collapses radially inwards to the single point at the center, so that a resulting space-time catastrophe there is perhaps not surprising. Could not the presence of perturbations which destroy the spherical symmetry alter the situation drastically? The recent rotating solution of Kerr[3] also possesses a physical singularity, but since a high degree of symmetry is still present (and the solution is algebraically special), it might again be argued that this is not representative of the general situation.[4] Collapse without assumptions of symmetry[5] will be discussed here.

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- Such a term **did not existed** at this time! It was introduced by J. A. Wheeler in 1967 only (see the book by Kip Thorne, 'Black holes and time warps' (1995), pp. 256-257).
- The term is used in the title of Steven Hawking's 1972 paper: **Black holes in General Relativity**, Commun. Math. Phys. **25**, 152-166. The above mentioned **Penrose**'s paper about singularities, **Penrose**'s 1968 paper 'Structure of space-time', In Batelle Rencontres, C. M. de Witt and J. A. Wheeler, eds. (Benjamin, New York), and **Hawking**'s black holes paper, put the theory of black hole physics at a solid ground. Perhaps, if Steven Hawking was alive, he would also be mentioned in this year Nobel Prize.

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In the 1965 Phys. Rev. Lett. paper Penrose formulates and sketches the proof of the theorem which, in a bit modernized version, reads as follows.

**Theorem**

If the space-time contains a **non-compact Cauchy hypersurface** and **a closed future-trapped surface**, and if **the convergence condition** $Ric(u, u) \geq 0$ **holds for null vectors** $u$ , then **there are future incomplete null geodesics.**

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The **Nobel Prize in Physics** 2020 was divided,

- one half awarded to **Roger Penrose** “for the discovery that black hole formation is a robust prediction of the general theory of relativity”,
- the other half ...

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a 4-dimensional manifold $M$ equipped with a *metric* $g$ of *Lorentzian signature* $(-,+,+,+)$,
- points of $M$ – are **physical events**; curves in $M$ – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors $u$ always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
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- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons$\sim$ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors $u$ satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or $0$ (for masless particles as e.g. photons).

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- Thus, the movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection $\nabla$ of the metric $g$, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equiations**
$$Ric - \frac{1}{2}Sg + \Lambda g = \kappa T,$$
where $\Lambda$ is a (cosmological) constant, $\kappa$ is a universal constant, $Ric$ is the Ricci tensor of $g$, $S$ is its Ricci scalar, and $T$ is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once $g$ satisfying Einstein's equations is given in $M$, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position $p$ and velocity $u$ of a particle at $p$, follow a geodesic passing through $p$ and tangent to $u$; this is the worldline of the considered particle.

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- the simplest spacetime is the **Minkowski spacetime**, with $M = \mathbb{R}^4$ and $g = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$; here $(t, x, y, z)$ are the Cartesian coordinates in $\mathbb{R}^4$; in this spacetime the geodesics are straight lines, and thus free particles' worldlines are timelike straight lines; **no gravitation**, as the curvature of the metric **Riemann**$\equiv 0$;

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- one of the the simplest **non-flat** metrics is **Schwarzschild's metric**,
  $$g = -(1-\frac{2m}{r})\mathrm{d}t^2 + \frac{1}{1-\frac{2m}{r}}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2);$$
  It is interpreted as a metric of a **spacetime outside a spherically symmetric mass** $m$ centered at $r=0$. It is a **solution to the Einstein's equations** with $T=0$ and $\Lambda=0$; It was obtained by **Karl Schwarzschild** in 1915 under the assumption that the metric is **spherically symmetric**; The metric has **non**vanishing Riemann tensor; nowadays we say that the Schwarzschild metric describes gravitational filed of the **most general** stationary, **non**rotating **black hole**;
- The metric $g$ was obtained by Schwarzschild for the region $r>2m$. Note that apart from spherical symmetry, the metric $g$ has also a Killing vector $\partial_t$; Thus, in the region considered by Schwarzschild, the metric/corresponding spacetime is **stationary** (even static).

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- as first explained by Eddington (1924) and Lemaître (1933), the singularity of the metric at $r = 2m$ corresponds to a **coordinate singularity**; for example the substitution $-\mathrm{d}t + \mathrm{d}r/(1 - 2m/r) \to \mathrm{d}v$ brings the Shwarzschild metric to the form
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- Nevertheless since $g(\partial_t, \partial_t) = -(1 - \frac{2m}{r})$, the **surface** $r = 2m$ **is interesting**: for $r > 2m$ the **Killing vector** $\partial_t$ is timelike - the **spacetime is stationary** in this region, for $r = 2m$ it is optical, and for $r < 2m$ its is **spacelike** - the spacetime is **NOT** stationary there. In particular, the area function $4\pi r^2$ of round spheres of radius $r$ in the region $r < 2m$ becomes **timelike** coordinate there! Strange, ha?

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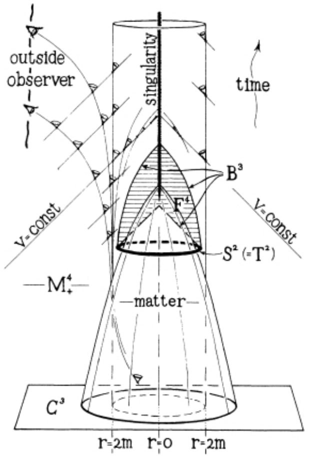
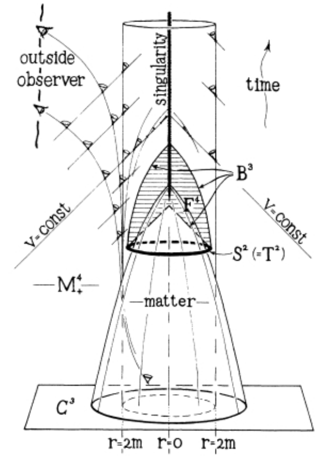



FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

# Shwarzschild black hole as a result of spherically symmetric collapse

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## Critics: singulsarities are artefacts of symmetry

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# Critics: singulsarities are artefacts of symmetry

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- **Theorem** (Raychaudhuri 1955, Komar 1956)
  Assume $\Lambda = 0$ and a perfect fluid energy momentum tensor $T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$, $u_\mu u^\nu = -1$, whose velocity vector field $u^\mu$ is **geodesic** and **irrotational**. If the expansion $\theta$ of $u$ is positive (negative) at an instant of time, and if $Riv(u, u) \geq 0$, then the energy density $\rho$ of the fluid diverges in the finite past (future) along every integral curve of $u^\mu$.
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- A: Raychaudhuri/Komar theorems are about (important) but particular form of matter content of spacetime – the **perfect fluid**. They connect energy condition $Ric(u, u) \geq 0$ with a vector field $u$ comoving with the fluid. If there was no this connection the focusing property of $u$ predicted by the Raychaudhuri equation would talk about e.g. caustics, and not physical singularity. Also the theorem is valid for surface-forming $u$s only.
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- Q: Why the Penrose 1965 theorem

  > *If the space-time contains a* **non-compact Cauchy hypersurface** *and* **a closed future-trapped surface**, *and if* **the convergence condition** $Ric(u,u) \geq 0$ **holds for null vectors** $u$, *then* **there are future incomplete null geodesics**

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- A: Raychaudhuri/Komar theorems are about (important) but particular form of matter content of spacetime – the **perfect fluid**. They connect energy condition $Ric(u,u) \geq 0$ with a vector field $u$ comoving with the fluid. If there was no this connection the focusing property of $u$ predicted by the Raychaudhuri equation would talk about e.g. caustics, and not physical singularity. Also the theorem is valid for surface-forming $u$s only.
- Penrose has the energy condition $Ric(u,u) \geq 0$ for all null vectors $u$ in spacetime; we now understand the neccessity of this kind of condition.
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The situation is quite **different** beyond the Schwarzschild radius, $r < 2m$, in the Schwarzschild spacetime! It follows that every 2-sphere $r = r_0 < 2m$, $t = const$ there, has expansion $\theta < 0$ for **both** families of 'outward' and 'inward' optical rays, at every point!
Physically, the gravitational field inside the Schwarzschild radius is that strong, that even **outgoing light rays will converge**!
This results in a **Definition** of a **trapped** surface $\Sigma$ in **any** spacetime to be a **closed** surface such that **both** families of orthogonal optical directions emanating from it orthogonally have expansion $\theta < 0$ across $\Sigma$.

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# Spacetime singularity

The spacelike hypersurface $r = 0$ in Schwarzschild has the property, such that the worldlines of all dust particles forming a collapsing star finish their life there. They happily follow their timelike goedesics, and **suddenly** in their **finite** time they die. They die because the **spacetime ends**! Their **internal clock** dies and its hands can not move anymore!

This motivates Penrose's **definition of singularity**:

Spacetime is **singular** if it is **geodesically incomplete**. More precisely: it is singular if **there is at least one incomplete causal geodesic in it**. And, an affinely parmetrized geodesic is **incomplete** if one can **not** prolong its affine parameter up to $+\infty$ (future incomplete), and/or $-\infty$ (past incomplete).

Of course such situation happens for the star's surface particles in an Oppenheimer-Snyder collapse: they die at the Schwarzschild singularity at $r = 0$; their world lines can not be prolonged in their proper time beyond this surface.

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So here is again **Penrose's Nobel Prize winning theorem**:

> *If the space-time contains a* **non-compact Cauchy hypersurface** *and* **a closed future-trapped surface**, *and if* **the convergence condition** $Ric(u, u) \geq 0$ **holds for null vectors** $u$, *then* **there are future incomplete null geodesics**.

We now understand every bit of it! Even if the collapse is not spherically symmetric, a small continuous deformation of this symmetry will **not destroy** the existence of **the trapped surface** and the **singularity** (in the spirit of Penrose's definition) **will be formed**!

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But...

**Hawking**: If the collapse is exactly **spherically symmetric** the metric is that of Schwarzschild solution outside the star, and has the following properties:

- The surface of the star will pass inside the Schwarzschild radius $r = 2m$. After this has happened there will be **closed trapped surface** around the star.
- There is a spacetime singularity.
- The singularity is **not visible** to observers who remain outside the Schwarzschild radius. This means that the **breakdown of our present physical theory** which one expects to occur at a singularity **cannot affect what happens outside the Schwarzschild radius** and **one can still predict the future in the exterior region** from Cauchy data on a spacelike surface.

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- If the parameter $a > m$ in the Kerr solution, i.e. if the angular momentum of the black hole is too large, the Kerr's singularity is **not hidden under the horizon**, it is **visible** by a distant observer. So the third property of Schwarzschild collapse is not true for the **exact solution of the vacuum Einstein's equations** given by Kerr, when the angular momentum is too large.
- **Penrose conjectures** then, that **black holes** such as Kerr's **with too large angular momentum can not be formed in reality**. There is a **cosmic censor** that prevents it.
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