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## DISQUOTATIONAL TRUTH AND ANALYTICITY

VOLKER HALBACH

**Abstract.** The uniform reflection principle for the theory of uniform T-sentences is added to PA. The resulting system is justified on the basis of a disquotationalist theory of truth where the provability predicate is conceived as a special kind of analyticity. The system is equivalent to the system ACA of arithmetical comprehension. If the truth predicate is also allowed to occur in the sentences that are inserted in the T-sentences, yet not in the scope of negation, the system with the reflection schema for these T-sentences assumes the strength of the Kripke–Feferman theory KF, and thus of ramified analysis up to  $\varepsilon_0$ .

**§1. Disquotational truth.** According to disquotationalism, the meaning of the truth predicate is governed by the (local) *disquotation sentences*:

$$(DS) \quad T^\Gamma \phi^\neg \leftrightarrow \phi.$$

Because of the liar paradox, only sentences  $\phi$  not containing the truth predicate T are allowed in the disquotation scheme.

One can strengthen the disquotation sentences by requiring their uniformity. I call these stronger variants the *uniform disquotation sentences*:

$$(UDS) \quad \forall x (T^\Gamma \phi(\dot{x})^\neg \leftrightarrow \phi(x)).$$

The dot above  $x$  is used, as usually, for indicating that the numeral for  $x$  is formally substituted for the free variable  $x$ .

While the local disquotation sentences yield a disquotational theory of truth (considered as a unary predicate), their uniform counterparts yield a disquotational theory of *satisfaction*, where satisfaction is a relation between formulas  $\phi(v)$  and objects  $x$ . As I am dealing exclusively with arithmetic, truth may be conceived as a unary predicate because closed terms (numerals) are available for all numbers.

Even if the disquotation sentences, or their uniform strengthenings, are combined with the axioms of PA or a similar theory, they are disappointingly weak; they do not establish any new mathematical insights. Tarski [30] observed the deductive weakness of the disquotation sentences as axioms. Philosophers express this weakness by saying that the disquotation sentences do not prove infinite ‘generalizations.’ Since many disquotationalists claim that the expression of infinite conjunctions is

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the only purpose of truth, this means that a truth predicate axiomatized by the disquotation sentences cannot serve this purpose, at least if 'expressing' is understood as 'proving' (cf. also Field [10] and Halbach [16]).

Typical generalizations are sentences of the form  $\forall x(\phi(x) \rightarrow Tx)$ , where  $\phi(x)$  defines some set of sentences. For instance, one may think of  $\phi(x)$  as saying that  $x$  is a propositional tautology, or a theorem of a particular theory. The sentence  $\forall x(\phi(x) \rightarrow Tx)$  is then conceived as the generalization of all sentences  $\phi(\ulcorner \theta \urcorner) \rightarrow \theta$  where  $\theta$  ranges over all sentences of  $\mathcal{L}_{PA}$ . The following obvious proposition shows that DS and UDS do not allow for the proof of any generalization we cannot already make without the truth predicate.

**PROPOSITION 1.1.** (i) *Assume  $PA+DS \vdash \forall x(\phi(x) \rightarrow Tx)$ . Then there are sentences  $\psi_1, \psi_2, \dots, \psi_k$  such that*

$$PA \vdash \forall x(\phi(x) \rightarrow x = \ulcorner \psi_1 \urcorner \vee x = \ulcorner \psi_2 \urcorner \vee \dots \vee x = \ulcorner \psi_k \urcorner).$$

*That is, PA proves that  $\phi(x)$  defines a finite set of sentences.*

(ii) *If  $PA+UDS \vdash \forall x(\phi(x) \rightarrow Tx)$ , then there is a number  $n$  such that*

$$PA \vdash \forall x(\phi(x) \rightarrow Tr_n(x))$$

*holds. The formula  $Tr_n(x)$  is a partial definition of truth for sentences with at most  $n$  logical symbols.*

The proposition remains valid if all induction axioms containing the truth predicate are allowed in  $PA+DS$  and  $PA+UDS$ .

**PROOF.** (i) Given a proof in  $PA+DS$ , pick as  $\psi_1, \psi_2, \dots, \psi_k$  those sentences  $\psi$  such that  $T\ulcorner \psi \urcorner \leftrightarrow \psi$  occurs as an axiom in the proof. To see that PA must prove  $\forall x(\phi(x) \rightarrow x = \ulcorner \psi_1 \urcorner \vee \dots \vee x = \ulcorner \psi_k \urcorner)$ , replace the atomic formulas  $Tt$  ( $t$  a closed term) everywhere in the proof by  $t = \ulcorner \psi_1 \urcorner \vee \dots \vee t = \ulcorner \psi_k \urcorner$ .

Clause (ii) can be proved similarly. The truth predicate can be replaced everywhere in a given proof by a suitable partial truth predicate.  $\dashv$

Thus, those generalizations provable in DS are always only generalizations over finitely many sentences. Generalizations provable from  $PA+UDS$  concern only sentences of limited logical complexity. The relevant generalization requires only a partial truth predicate, which is definable in  $\mathcal{L}_{PA}$ .

Proposition 1.1 is provable in PA. Hence, one can see from the standpoint of PA that the truth predicate of UDS is superfluous for proving generalizations. The truth predicate necessary for the generalizations that can be proved in  $PA+UDS$  can be defined within PA, and neither new predicates nor axioms are required for these generalizations.

The following well-known result is also a consequence of the proof of the above proposition: adding all disquotation sentences DS or UDS to PA, and extending the induction scheme of PA to the new language with the truth predicate, yields a theory that is conservative over PA (see, e.g., Halbach [15] and Ketland [19] for a philosophical evaluation).

In contrast, more sophisticated theories of truth do prove important generalizations not provable in PA, such as "all theorems of PA are true," and thus these theories are stronger than PA. Adding the 'inductive' axioms for satisfaction to PA yields the system  $PA(S)$  (defined below), which is equivalent to the system ACA of

arithmetical comprehension if induction axioms with T are allowed. Some philosophers have emphasized that a so-called ‘deflationist’ truth predicate must satisfy the ‘inductive’ axioms; otherwise, it could not serve its purpose (see Shapiro [27], Field [10] and Halbach [17]). These axioms allow for the proof of generalizations and they imply all uniform disquotation sentences, but the ‘inductive’ axioms do not follow conversely from the sentences UDS.

The ‘inductive’ axioms are no longer disquotational. Thus, it seems that disquotationalism fails because its ‘meaning postulates’ for truth (that is, the disquotation axioms) do not yield a sufficiently strong theory of truth. It seems that truth theories based on non-disquotationalist axioms are superior, and that only truth predicates axiomatized by these stronger theories allow the truth predicate to serve its purpose; namely, making generalizations. For the disquotation sentences are simply too weak for generalizations.

The lack of deductive power is the less satisfying since logicians have devised systems of truth exceeding by far even the proof-theoretic strength of ACA ([2, 6, 11]). In these systems, truth applies provably to sentences that contain the truth predicate. They axiomatize different ‘solutions’ of the liar paradox (for instance, Kripke’s [21] theory of truth). Nothing like this has been available for the disquotational conception of truth. Hence, it is not surprising that the disquotational conception of truth has been neglected by logicians.

I shall show that the disquotational theories of truth are not necessarily deductively weak. A disquotational theory of truth can serve the purpose of proving non-trivial generalizations. I deny that the disquotation sentences exhaust the disquotational standpoint. I shall propose axioms that are still disquotational, but are nevertheless strong enough to imply the ‘inductive’ axioms for truth.

**§2. Analyticity.** In this paper, I propose to overcome the lack of deductive power of disquotational theories of truth by paying attention to the status of the disquotation sentences. Since the disquotation sentences govern the meaning of the truth predicate, they are *analytic*, according to the disquotationalist account.

Roughly speaking, Carnap [4] proposed an explication of analyticity in terms of the equation

$A$  is analytic iff  $A$  is implied by the meaning postulates.

More precisely, on Carnap’s view,  $A$  is analytic if and only if  $A$  follows (semantically) in first-order logic from the meaning postulates. Although Carnap had semantical consequence in mind, one may replace semantical consequence by provability in first-order logic in this explication of analyticity, because in the realm of first-order logic adequate deductive systems are available.

Carnap’s analysis of general analyticity has encountered serious criticism for several reasons. The most momentous attack was launched by Quine [25]. Quine argued that there is no fixed set of meaning postulates for an arbitrary given term.<sup>1</sup> This concerns analytic sentences like “All bachelors are unmarried,” because it is quite unclear what the meaning postulates for “bachelor” and “unmarried” may be.

I agree with Quine that this is a problem for the general notion of analyticity.

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<sup>1</sup>Quine took offense at Carnap’s analysis also for other reasons. Here, I am not able to discuss the controversy in any detail. For a recent and more elaborate view on the controversy see [12].

Certain restricted notions of analyticity, however, are not affected by Quine's objections against general analyticity. In particular, we might have a precisely determined set of meaning postulates for a particular notion, and thereby escape Quine's criticism. This holds in particular in the case of disquotational truth: disquotational truth is characterized by the fact that its meaning is governed by the disquotation sentences. Thus, the doctrine of disquotationalism may be parsed as the claim that the disquotation sentences form the set of meaning postulates for truth.

Therefore, I propose the following explication of analyticity in the (disquotational) truth predicate (or of truth-analyticity, for short):

*A* is truth-analytic iff *A* is logically implied by the disquotation sentences.

A sentence is logically implied by the disquotation sentences if and only if it follows from them in first-order logic with identity.

Of course, one may set up axioms for a symbol in any arbitrary way and pronounce them as meaning postulates for a certain concept expressed by that symbol. For instance, a set-theoretic platonist can lay down the axioms of ZFC as meaning postulates for set-theoretic membership, and claim that the consequences of these axioms are analytic in  $\in$ . Even a 'finitist' who has adopted primitive recursive arithmetic as his standpoint, and is not willing to go beyond PRA, has the concept of provability from the ZFC axioms, and he can agree that they govern the platonist's concept of membership.<sup>2</sup> In contrast to the platonist, however, the finitist will not believe in the soundness of the concept of set-theoretic membership, although he can talk about provability in ZFC.

Similarly, the concept of provability from the disquotation sentences is available also to those rejecting disquotationalism. Calling sentences provable from DS, or UDS, truth-analytic is just a terminological decision. What sets the disquotationalist apart is that he believes in the soundness of his concept of truth.

The soundness of an axiom system can be expressed by reflection principles or reflection rules. These allow passage from the provability of a formula to the formula itself.

The reflection principles for the disquotation sentences DS are a proper extension of a theory of syntax like PA:

PROPOSITION 2.1. *The system PA, formulated in the language  $\mathcal{L}_T$ , and with all induction axioms in the language  $\mathcal{L}_T$ , does not prove all instances of the reflection axiom*

$$\forall x (\text{Bew}_{\text{DS}}(\ulcorner \phi(x) \urcorner) \rightarrow \phi(x)).$$

Here,  $\text{Bew}_{\text{DS}}(v)$  designates provability from the disquotation sentences.

The claim follows from Proposition 9.2 below.

The same holds if  $\text{Bew}_{\text{DS}}(v)$  is understood as provability from the *uniform* disquotation sentences.

Proposition 2.1 holds despite the fact that PA proves the consistency of DS and UDS, and their conservativeness over weak theories of arithmetic (see Halbach [17]).

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<sup>2</sup>I do not want to claim that PRA is the correct theory for finitistic reasoning. I am just using 'finitist' as a handy designation for somebody accepting the axioms of PRA as his mathematical standpoint.

Since the trust in the concept of disquotational truth can be expressed by a reflection scheme like the one in Proposition 2.1, I propose to formalize the disquotationalist standpoint by such reflection principles. As I will show in §8, reflection *rules* could be used as well without any significant loss in deductive power.

One might suspect that Proposition 2.1 poses a problem for the disquotationalist. Does the proposition show that the disquotationalist's set of meaning postulates lacks something? Should not a reflection principle be included along with the disquotation sentences in the set of meaning postulates for truth? No, it should not: the reflection axioms are, at most, meaning postulates for truth-*analyticity*, but not for truth *itself*. A full account of disquotational truth is afforded by the disquotation sentences.

The formalization of the disquotationalist standpoint by a reflection scheme takes into account that the disquotationalist does not only claim the disquotation sentences, but that he also claims something about them, namely, that they govern the meaning of the truth predicate.

In this respect, disquotationalism is different from most mathematical standpoints: a set theorist usually claims only certain set-theoretic principles, e.g., the axioms of ZFC and perhaps some large cardinal axioms, or some combinatorial principles. However, on the traditional account at least, he will not say anything about the modal status of these axioms.<sup>3</sup>

In general, I do not propose that axioms of a theory should be replaced by the uniform reflection principles for that theory. For the transition from a theory to a reflection principle for that theory requires an argument. In the case of disquotationalism this is provided by paying attention to the modal status of the disquotation sentences, i.e., by arguments for their analyticity.

There is a family of arguments purporting to show that the disquotation sentences are neither analytic nor necessary, because the meanings of linguistic expressions are purely conventional and thus only contingent.

For instance, "Snow is white" could have meant that grass is white. In this case, the left-hand side of the disquotation sentence

"Snow is white" is true if and only if snow is white

is false, while the right-hand side continues to be true, because the color of snow is not affected by semantical facts.

If these arguments were sound they would undermine the justification of the reflection principle for DS (without refuting the disquotation sentences themselves). The modal status of the disquotation sentences is the moot point between deflationists (disquotationalists) and the proponents of a substantive account of truth.

In particular, Field [7] has argued that the above kind of reasoning does not apply to disquotational truth. "Snow is white" would continue to be true in the disquotational sense even if we were using words in a different way.

Here, I do not try to investigate this topic, because it requires a detailed discussion. I claim only that the reflection principle for DS and UDS captures the disquotationalist standpoint, whether or not it is correct. I do not wish to undertake a defense of the disquotationalist standpoint itself. Defenses of disquotationalism

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<sup>3</sup>Perhaps the set theorist can arrive at a justification of proof-theoretic, or even set-theoretic, reflection principles by mathematical reasons, but this is another story.

are provided, e.g., in Field [8, 9] and Halbach [17]. I have also defended the account of truth-analyticity advanced here in [16].

The idea of strengthening disquotational truth theories by claiming that the disquotation sentences (or similar principles) are necessary, analytic, or the like, is not new. Proposals in this direction have been made by Sosa [28] and McGrath [24]. Their approach differs significantly from my account with respect to the philosophical assumptions made. In [13], I applied a similar technique as in the proof of Lemma 4.2 below; in the proof-theoretic analysis of a system FS of self-referential truth, I have exploited the *truth* (rather than the necessity) of the disquotation sentences in order to embed a system of ramified truth in FS.

I shall now give a short overview of the remaining sections of the paper.

The following §3 contains some preliminaries. In §4, I shall investigate the system AT (A for “analyticity” and T for “truth”) obtained by adjoining certain reflection principles, like the one in Proposition 2.1, to PA. Instead of the local disquotation sentences, I shall employ uniform disquotation; the resulting system is proof-theoretically equivalent to the system PA(*S*) of ‘inductive’ truth, and to the system ACA of arithmetical comprehension.

By relaxing the strict object-/metalanguage distinction in the disquotation sentences in a straightforward way, the strength of the system can be further increased.

A main ingredient for the liar paradox is *negation*. Therefore, those instances of the uniform disquotation scheme UDS where  $\phi(x)$  does not contain an occurrence of T in the scope of a negation symbol should be safe from inconsistency. This is a crude method for avoiding paradox; but it suffices for strengthening AT considerably. If the uniform reflection principle for the extended uniform disquotation scheme is employed, then the resulting system AT<sup>+</sup> attains the strength of a comparably strong truth theory, namely, the Kripke–Feferman theory KF, which is known to be equivalent to ramified analysis up to  $\varepsilon_0$ . The system AT<sup>+</sup> is studied in §§5–7.

In §8, some modifications of the main results are considered. Some open questions with respect to weakened versions of the system AT<sup>+</sup> are discussed in the final §9.

**§3. Arithmetization of syntax.** For the following, I need to fix some notation. The logical symbols of the language  $\mathcal{L}_{PA}$  of arithmetic are  $\neg$ ,  $\vee$ ,  $\wedge$ ,  $\forall$  and  $\exists$ . For the sake of simplicity I assume that  $\mathcal{L}_{PA}$  does not contain any function symbols besides the constant for 0 and the symbol for the successor function. Therefore numerals, i.e., the constant for 0 preceded by a string of successor symbols, are the only closed terms of  $\mathcal{L}_{PA}$ . In order to make the notation more perspicuous, I shall write formulas as if other function expressions were available. Formulas of  $\mathcal{L}_{PA}$  involving such function expressions must be thought of as shorthand for corresponding formulas where the function expressions have been eliminated by predicate expressions.

I do not distinguish expressions from their respective Gödel numbers, in order to render the following more readable.

$\forall \phi \in \mathcal{L}_{PA} \dots$  is short for  $\forall x (\text{Sent}_{\mathcal{L}_{PA}}(x) \rightarrow \dots)$  where  $\text{Sent}_{\mathcal{L}_{PA}}(x)$  says that  $x$  is an  $\mathcal{L}_{PA}$ -sentence. Since I want to avoid awkward notation with many function

expressions that need to be explained, I write, e.g.,

$$\forall \phi \in \mathcal{L}_{PA} \text{ T}^{\neg\phi^{\neg}}$$

in order to express that all negations of  $\mathcal{L}_{PA}$ -sentences are true. Of course,  $\neg\phi^{\neg}$  is not a numeral, but rather a complex term with a free variable. The term is then eliminated by suitable predicate expressions.

**§4. Typed disquotation.** In systems of typed truth, the truth of a sentence containing the (same) truth predicate cannot be proved. I propose a system of disquotational truth based on UDS. Since in the disquotational sentences of UDS only sentences not containing the truth predicate are allowed as instances, UDS yields a typed notion of truth, and the system I shall present in the present § is typed as well.

More precisely, I employ a system embracing reflection axioms for truth-analyticity, i.e., for UDS:

**DEFINITION 4.1.** The system AT is PA with induction in  $\mathcal{L}_T$ , expanded by the following additional axioms:

$$\forall \vec{x} (\text{Bew}_{\text{UDS}}(\text{T}^{\neg\phi(\vec{x})^{\neg}}) \rightarrow \phi(\vec{x})).$$

$\text{Bew}_{\text{UDS}}(v)$  expresses provability from the uniform disquotational sentences;  $\vec{x}$  is a finite string of variables. The formula  $\phi(\vec{v})$  is an arbitrary formula; in particular, it may contain the truth predicate T. The system AT embraces also all induction axioms, including those containing T.

The theory UDS does not have any arithmetical axioms; hence it cannot code pairs. This is the reason for formulating the reflection axioms with more than one variable. In practice only two variables will be needed.

**LEMMA 4.2.** *The system AT proves all 'inductive' clauses for truth.*

By the inductive clauses I mean the following sentences:

- (1)  $\forall x, y (\text{T}^{\neg x = y^{\neg}} \leftrightarrow x = y)$
- (2)  $\forall \phi \in \mathcal{L}_{PA} (\text{T}^{\neg\neg\phi^{\neg}} \leftrightarrow \neg\text{T}^{\neg\phi^{\neg}})$
- (3)  $\forall \phi, \psi \in \mathcal{L}_{PA} (\text{T}^{\neg\phi \wedge \psi^{\neg}} \leftrightarrow (\text{T}^{\neg\phi^{\neg}} \wedge \text{T}^{\neg\psi^{\neg}}))$
- (4)  $\forall \phi, \psi \in \mathcal{L}_{PA} (\text{T}^{\neg\phi \vee \psi^{\neg}} \leftrightarrow (\text{T}^{\neg\phi^{\neg}} \vee \text{T}^{\neg\psi^{\neg}}))$
- (5)  $\forall \phi(v) \in \mathcal{L}_{PA} (\text{T}^{\neg\forall v \phi(v)^{\neg}} \leftrightarrow \forall x \text{T}^{\neg\phi(x)^{\neg}})$
- (6)  $\forall \phi(v) \in \mathcal{L}_{PA} (\text{T}^{\neg\exists v \phi(v)^{\neg}} \leftrightarrow \exists x \text{T}^{\neg\phi(x)^{\neg}})$

**PROOF.** As an example I treat the universal quantifier case (5). Only the quantifier axioms (5) and (6) require reflection for the *uniform* disquotational sentences;



otherwise reflection for DS suffices.

- (7)  $\forall \phi(v) \in \mathcal{L}_{PA} \text{ Bew}_{UDS}(\ulcorner \forall v (\text{T}^\ulcorner \phi(v) \urcorner \leftrightarrow \phi(v)) \urcorner)$
- (8)  $\forall \phi(v) \in \mathcal{L}_{PA} \text{ Bew}_{UDS}(\ulcorner \forall v \text{T}^\ulcorner \phi(v) \urcorner \leftrightarrow \forall v \phi(v) \urcorner)$  (7)
- (9)  $\forall \phi(v) \in \mathcal{L}_{PA} \text{ Bew}_{UDS}(\ulcorner \text{T}^\ulcorner \forall v \phi(v) \urcorner \leftrightarrow \forall v \phi(v) \urcorner)$  def. of UDS
- (10)  $\forall \phi(v) \in \mathcal{L}_{PA} \text{ Bew}_{UDS}(\ulcorner \text{T}^\ulcorner \forall v \phi(v) \urcorner \leftrightarrow \forall v \text{T}^\ulcorner \phi(v) \urcorner \urcorner)$  (8) and (9)
- (11)  $\forall \phi(v) \in \mathcal{L}_{PA} \text{ Bew}_{UDS}(\ulcorner \text{T}^\ulcorner \forall v \phi(v) \urcorner \leftrightarrow \forall x \text{T}^\ulcorner \phi(x) \urcorner \urcorner)$  renaming
- (12)  $\forall \phi(v) \in \mathcal{L}_{PA} (\ulcorner \text{T}^\ulcorner \forall v \phi(v) \urcorner \leftrightarrow \forall x \text{T}^\ulcorner \phi(x) \urcorner \urcorner)$

A reflection axiom of AT is used for the derivation of the last line from (11).

In the step from (10) to (11), where the variable is renamed, it is assumed that there is no variable collision. The renaming is necessary because  $v$  may be a nonstandard variable (when AT is interpreted by a nonstandard model), while  $x$  is not only mentioned but actually used, and thus has to be a standard variable.  $\dashv$

AT can prove generalizations not provable from the uniform disquotation sentences: by the lemma, AT is as good at this as the well-established truth theory  $PA(S)$  which has been investigated by model and proof theorists in detail (see [20, 6]). The theory  $PA(S)$  is given by the inductive clauses (1)–(4) and the axioms of PA. For instance, AT can prove that all theorems of PA are true, while the single instance  $\text{Bew}(\ulcorner 0 = 1 \urcorner) \rightarrow 0 = 1$  is not provable in PA by Gödel’s second incompleteness theorem.

$PA(S)$  has been proposed for overcoming the deductive weakness of theories like  $PA+UDS$  (see, e.g., [27]), because  $PA(S)$  proves important non-trivial generalizations. As far as I can see, it is also complete as a theory of arithmetical truth in the sense that all truth-theoretic principles follow from  $PA(S)$ . Of course  $PA(S)$  is a recursively enumerable system and therefore not plainly complete. However, all sentences independent from  $PA(S)$  do not seem to be justifiable on purely truth-theoretic reasons (if attention is restricted to truth of arithmetical sentences). For instance, certain combinatorial principles, and the consistency statement for  $PA(S)$  itself, are independent of  $PA(S)$ . However, as far as I can see, they cannot be justified by reflecting on arithmetical truth.

Be this as it may, AT is at least as ‘complete’ as  $PA(S)$ , because  $PA(S)$  is a subtheory of AT by Lemma 4.2.

Conversely, AT is not stronger than  $PA(S)$ .

**THEOREM 4.3.** *The systems AT,  $PA(S)$  and ACA are  $\mathcal{L}_{PA}$ -conservatively interpretable in one another.*

The equivalence of ACA and  $PA(S)$  is well known (see Feferman [6]). The system ACA defines a truth predicate satisfying the axioms of  $PA(S)$  (see [29, p. 183], [14, Section 10]).

$PA(S)$  is a subtheory of AT by the preceding lemma. The reduction of AT to  $PA(S)$  is akin to the argument in §7. One proves that there is a finite subtheory of  $PA(S)$  which proves all axioms of UDS. That AT is a subtheory of  $PA(S)$  follows then from the uniform reflexivity of  $PA(S)$ .

**§5. Untyped disquotation.** Theorem 4.3 shows that the disquotational conception can catch up with the ‘inductive’ conception of truth with respect to deductive power. AT—and thus  $PA(S)$ —is a system of typed truth, because truth is applied only to T-free sentences, in the sense that one cannot prove in AT, or  $PA(S)$ , the truth of a sentence containing T.

As pointed out above, several authors have proposed systems that naturally generalize  $PA(S)$  by relaxing the object-/metalanguage distinction. In untyped theories, truth is axiomatized in such a way that it provably applies also to sentences containing the truth predicate.

The resulting systems have proved useful and mathematically fertile. Moreover, several applications have been suggested (see Cantini [3]).

One important specimen of such a system is the Kripke–Feferman system KF. It relates to Kripke’s [21] fixed-point semantics with the strong Kleene evaluation scheme in the same way as  $PA(S)$  relates to Tarski’s truth definition. In both cases, the clauses of an inductive definition of truth are turned into axioms.

KF employs an additional predicate F for falsity. The language with T and F is called  $\mathcal{L}_{T,F}$ . Beyond the axioms of PA, KF embraces all induction axioms in the language with the truth predicate, and the following truth-theoretic principles:

- (KF1)  $\forall x, y (T \ulcorner \dot{x} = \dot{y} \urcorner \leftrightarrow x = y)$
- (KF2)  $\forall x, y (F \ulcorner \dot{x} = \dot{y} \urcorner \leftrightarrow \neg x = y)$
- (KF3)  $\forall \phi \in \mathcal{L}_{T,F} (T \ulcorner \neg \phi \urcorner \leftrightarrow F \ulcorner \phi \urcorner)$
- (KF4)  $\forall \phi \in \mathcal{L}_{T,F} (F \ulcorner \neg \phi \urcorner \leftrightarrow T \ulcorner \phi \urcorner)$
- (KF5)  $\forall \phi \in \mathcal{L}_{T,F} (T \ulcorner T \ulcorner \phi \urcorner \urcorner \leftrightarrow T \ulcorner \phi \urcorner)$
- (KF6)  $\forall \phi \in \mathcal{L}_{T,F} (F \ulcorner T \ulcorner \phi \urcorner \urcorner \leftrightarrow F \ulcorner \phi \urcorner)$
- (KF7)  $\forall \phi \in \mathcal{L}_{T,F} (T \ulcorner F \ulcorner \phi \urcorner \urcorner \leftrightarrow F \ulcorner \phi \urcorner)$
- (KF8)  $\forall \phi \in \mathcal{L}_{T,F} (F \ulcorner F \ulcorner \phi \urcorner \urcorner \leftrightarrow T \ulcorner \phi \urcorner)$
- (KF9)  $\forall \phi, \psi \in \mathcal{L}_{T,F} (T \ulcorner \phi \wedge \psi \urcorner \leftrightarrow (T \ulcorner \phi \urcorner \wedge T \ulcorner \psi \urcorner))$
- (KF10)  $\forall \phi, \psi \in \mathcal{L}_{T,F} (F \ulcorner \phi \wedge \psi \urcorner \leftrightarrow (F \ulcorner \phi \urcorner \vee F \ulcorner \psi \urcorner))$
- (KF11)  $\forall \phi, \psi \in \mathcal{L}_{T,F} (T \ulcorner \phi \vee \psi \urcorner \leftrightarrow (T \ulcorner \phi \urcorner \vee T \ulcorner \psi \urcorner))$
- (KF12)  $\forall \phi, \psi \in \mathcal{L}_{T,F} (F \ulcorner \phi \vee \psi \urcorner \leftrightarrow (F \ulcorner \phi \urcorner \wedge F \ulcorner \psi \urcorner))$
- (KF13)  $\forall \phi(v) \in \mathcal{L}_{T,F} (T \ulcorner \forall v \phi(v) \urcorner \leftrightarrow \forall y T \ulcorner \phi(\dot{y}) \urcorner)$
- (KF14)  $\forall \phi(v) \in \mathcal{L}_{T,F} (F \ulcorner \forall v \phi(v) \urcorner \leftrightarrow \exists y F \ulcorner \phi(\dot{y}) \urcorner)$
- (KF15)  $\forall \phi(v) \in \mathcal{L}_{T,F} (T \ulcorner \exists x \phi(x) \urcorner \leftrightarrow \exists y T \ulcorner \phi(\dot{y}) \urcorner)$
- (KF16)  $\forall \phi(v) \in \mathcal{L}_{T,F} (F \ulcorner \exists v \phi(v) \urcorner \leftrightarrow \forall y F \ulcorner \phi(\dot{y}) \urcorner)$

Different variants of the theory KF are found in the literature (see Reinhardt [26], Cantini [1] and McGee [23]). Most versions include a consistency axiom to the effect that no sentence is both true and false. Feferman’s [6] own variant  $\text{Ref}(PA(P))$  of KF lacks this axiom. The system KF without consistency axiom describes a version of Kripke’s theory that allows truth-value gluts as well as gaps (see Visser [31] for

this four-valued variant of Kripke's theory, and Cantini [1] for its relation to KF-like theories). The consistency axiom does not contribute anything to the proof-theoretic strength of KF. Moreover, it is very different from the other axioms KF1–KF16 because it is not a compositional axiom relating the truth of a more complex sentence to the semantic value of its constituents (see Feferman [6, p. 19]).

KF is much stronger than the typed theory  $PA(S)$  which KF generalizes. Roughly speaking, KF is equivalent to  $\varepsilon_0$ -many iterations of  $PA(S)$  (see Feferman [6]).

I shall show that the disquotational conception of truth is not necessarily surpassed by the inductive and compositional conception with respect to deductive power even in the type-free case (see Halbach [18] for an account of compositionality in the context of formal truth theories). In particular, I present a disquotational theory of truth as strong as KF.

How can one generalize AT and drop the type, or language-level, distinction? In order to obtain an untyped generalization of AT, we have to find an untyped variant of UDS. That is, we are seeking a theory of disquotation, not only for arithmetical sentences, but also for those containing the truth predicate. Finding meaning postulates for disquotational type-free truth comes down to generalizing the disquotation sentences UDS in a suitable way.

Clearly one cannot, upon pain of inconsistency, completely drop the restriction that  $\phi(x)$  must not contain the truth predicate in the uniform disquotation sentences. Yet there are well-known generalizations of the disquotation sentences not leading to inconsistency. I propose the following simple and straightforward extension of the uniform disquotation sentences.

DEFINITION 5.1. The system  $UDS^+$  is the set of all equivalences

$$\forall x (\ulcorner \phi(\dot{x}) \urcorner \leftrightarrow \phi(x))$$

where  $\phi(x)$  does not have an occurrence of the truth predicate in the scope of a negation symbol.

$AT^+$  is PA, expanded by the following reflection axiom:

$$\forall \vec{x} (\text{Bew}_{UDS^+}(\ulcorner \phi(\vec{x}) \urcorner) \rightarrow \phi(\vec{x})).$$

$\text{Bew}_{UDS^+}(\vec{x})$  expresses provability from  $UDS^+$ . The system  $UDS^+$  does not have any non-logical axioms besides the disquotation axioms.

$AT^+$  and KF 'almost' coincide.

THEOREM 5.2. *The systems  $AT^+$ , KF and  $RA_{<\varepsilon_0}$ , i.e., ramified analysis up to  $\varepsilon_0$ , are  $\mathcal{L}_{PA}$ -conservatively interpretable in one another.*

The theorem shows that the disquotational conception does not condemn truth to a role without any mathematical content; rather, disquotational truth is as useful as the truth predicate of the sophisticated, and clearly not disquotational, theory KF, which takes its plausibility from Kripke's fairly complicated inductive construction.

The rest of the paper is devoted to the proof of Theorem 5.2. The equivalence of KF and  $RA_{<\varepsilon_0}$  is due to Feferman [6]. It remains to show that  $AT^+$  and KF are equivalent.

**§6. Interpretation of  $AT^+$  in  $KF$ .** The system  $AT^+$  is a subtheory of  $KF$ . In the following it is shown that  $KF$  proves the uniform reflection scheme for  $UDS^+$  (Lemma 6.5).

The *T-positive* uniform disquotation sentences are defined in the following way:

**DEFINITION 6.1.** The truth predicate  $T$  occurs only positively in  $\phi \in \mathcal{L}_T$  if  $T$  occurs only in the scope of an even number of negation symbols. The *T-positive* uniform disquotation sentences are all sentences of the following form:

$$\forall x (T^\ulcorner \psi(\dot{x}) \urcorner \leftrightarrow \psi(x)),$$

where  $T$  occurs only positively in  $\psi(x)$ .

The *T-positive* uniform disquotation sentences can be established in  $KF$  by a meta-induction (see Cantini [1, Lemma 3.2 (ii)]). This does not require an instance of the induction scheme of  $KF$ . This observation yields the following lemma:

**LEMMA 6.2.** *There is a finite subtheory  $S_0$  of  $KF$  that proves all *T-positive* uniform disquotation sentences. This is provable in  $PA$ .*

The lemma is shown by a meta-induction on the complexity of formulas.

**COROLLARY 6.3.** *The system  $S_0$  proves all sentences in  $UDS^+$ . This is provable in  $PA$ .*

According to the next lemma,  $KF$  proves uniform reflection for  $S_0$ .

**LEMMA 6.4.** *The system  $KF$  is uniformly reflexive. That is, for any finite subtheory  $S \subset KF$ , the following holds:*

$$KF \vdash \forall \vec{x} (\text{Bew}_S(\ulcorner \phi(\vec{x}) \urcorner) \rightarrow \phi(\vec{x})).$$

The lemma is established in a similar way as the uniform reflexivity of  $PA$ . That is, one applies the formalized cut-elimination theorem for predicate logic, and then one employs partial truth predicates and proves a formalized soundness theorem. The additional predicate symbol  $T$  requires only an obvious modification of the partial truth predicates.

It follows that  $AT^+$  is a subtheory of  $KF$ :

$$\text{LEMMA 6.5. } KF \vdash \forall \vec{x} (\text{Bew}_{UDS^+}(\ulcorner \phi(\vec{x}) \urcorner) \rightarrow \phi(\vec{x})).$$

**PROOF.** By Corollary 6.3, we have for the set  $S_0$ , whose existence is proved there, the following:

$$KF \vdash \forall y (\text{Bew}_{UDS^+}(y) \rightarrow \text{Bew}_{S_0}(y)).$$

Therefore  $KF$  proves the uniform reflection principle for  $UDS^+$ , by Lemma 6.4.  $\dashv$

**§7. Interpretation of  $KF$  in  $AT^+$ .** The following lemma is trivial:

**LEMMA 7.1.** *The system  $AT^+$  proves all sentences in  $UDS^+$ .*

$\mathcal{L}_T^+$  is the sublanguage of  $\mathcal{L}_T$  that contains only those formulas of  $\mathcal{L}_T$  where  $T$  occurs only outside the scope of the negation symbol.

The definition of the auxiliary translation function  $I$  relies on the recursion theorem, because it is defined in terms of a representation  $I$  of  $I$  in  $\mathcal{L}_{PA}$  in the second and third line of the definition (see [14, Lemma 5.4]).

The expressions  $t$  and  $s$  are closed terms. They have to be numerals because, according to the conventions in §3,  $\mathcal{L}_T$  does not contain other closed terms.

$$I(\phi) := \begin{cases} \phi & \text{if } \phi = (s=t) \text{ or } \phi = (\neg s=t), \\ \mathbf{T}I(t) & \text{if } \phi = \mathbf{T}t \text{ or } \phi = \neg \mathbf{F}t, \\ \mathbf{T}I(\neg t) & \text{if } \phi = \neg \mathbf{T}t \text{ or } \phi = \mathbf{F}t, \\ I(\psi) & \text{if } \phi = \neg \neg \psi, \\ I(\psi) \wedge I(\chi) & \text{if } \phi = (\psi \wedge \chi), \\ I(\neg \psi) \vee I(\neg \chi) & \text{if } \phi = \neg(\psi \wedge \chi), \\ I(\psi) \vee I(\chi) & \text{if } \phi = (\psi \vee \chi), \\ I(\neg \psi) \wedge I(\neg \chi) & \text{if } \phi = \neg(\psi \vee \chi), \\ \forall x I(\psi) & \text{if } \phi = (\forall x \psi), \\ \exists x I(\neg \psi) & \text{if } \phi = (\neg \forall x \psi), \\ \exists x I(\psi) & \text{if } \phi = (\exists x \psi), \\ \forall x I(\neg \psi) & \text{if } \phi = (\neg \exists x \psi), \\ 0 = 0 & \text{else.} \end{cases}$$

The function  $J$  interpreting KF in  $\mathbf{AT}^+$  is defined as follows:

$$J(\phi) := \begin{cases} \phi & \text{if } \phi = (s=t) \text{ or } \phi = (\neg s=t), \\ \mathbf{T}I(t) & \text{if } \phi = \mathbf{T}t \text{ or } \phi = \neg \mathbf{F}t, \\ \mathbf{T}I(\neg t) & \text{if } \phi = \neg \mathbf{T}t \text{ or } \phi = \mathbf{F}t, \\ \neg J(\psi) & \text{if } \phi = \neg \psi, \\ J(\psi) \wedge J(\chi) & \text{if } \phi = (\psi \wedge \chi), \\ J(\psi) \vee J(\chi) & \text{if } \phi = (\psi \vee \chi), \\ \forall x J(\psi) & \text{if } \phi = (\forall x \psi), \\ \exists x J(\psi) & \text{if } \phi = (\exists x \psi), \\ 0 = 0 & \text{else.} \end{cases}$$

The reason why the interpretation function commutes with negation outside, but not within, the scope of  $\mathbf{T}$  is the difference between the external and internal logics of KF: the system KF is formulated in classical logic, that is, its external logic is classical, while its internal logic allows for truth-value gaps and gluts.

The following lemma is established by an obvious induction on the build-up of  $\mathcal{L}_{\mathbf{T},\mathbf{F}}$ -sentences.

**LEMMA 7.2.** *The system PA proves that  $J(\phi)$  is in  $\mathcal{L}_T^+$  for all  $\phi \in \mathcal{L}_{\mathbf{T},\mathbf{F}}$ .*

In order to show that  $\mathbf{AT}^+$  proves  $J(\phi)$  if KF proves  $\phi$ , one has to prove for any axiom of KF that its translation is a theorem of  $\mathbf{AT}^+$ .

The translation of the axiom KF1 is KF1 itself, which is a consequence of the uniform disquotatation sentences. Thus the claim follows from Lemma 7.1. The axiom KF2 is also handled easily.

KF3 and KF4 are rendered trivial theorems of  $\mathbf{AT}^+$  by the interpretation  $J$ .

From the group KF5–KF8 I treat only KF6. By Lemma 7.1 and Lemma 7.2, the following is provable in  $\mathbf{AT}^+$ :

$$\forall \phi \in \mathcal{L}_T \left( \mathbf{T} \ulcorner \mathbf{T} J(\ulcorner \neg \phi \urcorner) \urcorner \leftrightarrow \mathbf{T} J(\ulcorner \neg \phi \urcorner) \right).$$

By the definition of  $J$ , this implies the following sentence:

$$\forall \phi \in \mathcal{L}_T \left( J(\mathbf{F} \ulcorner \mathbf{T} \phi \urcorner) \leftrightarrow J(\mathbf{F} \ulcorner \phi \urcorner) \right).$$

The latter is obviously the translation of the axiom KF6.

The conjunction axiom KF9 can be dealt with in the following manner. By definition of  $\text{Bew}_{\text{UDS}^+}(v)$ , the following sentences are theorems of PA:

$$\begin{aligned} \forall \phi, \psi \in \mathcal{L}_T^+ \quad \text{Bew}_{\text{UDS}^+}(\ulcorner \text{T}^\ulcorner \phi \wedge \psi^\urcorner \leftrightarrow \phi \wedge \psi^\urcorner), \\ \forall \phi \in \mathcal{L}_T^+ \quad \text{Bew}_{\text{UDS}^+}(\ulcorner \text{T}^\ulcorner \phi^\urcorner \leftrightarrow \phi^\urcorner). \end{aligned}$$

The combination of both lines yields the following sentence:

$$\forall \phi, \psi \in \mathcal{L}_T^+ \quad \text{Bew}_{\text{UDS}^+}(\ulcorner \text{T}^\ulcorner \phi \wedge \psi^\urcorner \leftrightarrow \text{T}^\ulcorner \phi^\urcorner \wedge \text{T}^\ulcorner \psi^\urcorner \urcorner).$$

Applying the relevant reflection axiom yields the following sentence:

$$\forall \phi, \psi \in \mathcal{L}_T^+ \quad (\text{T}^\ulcorner \phi \wedge \psi^\urcorner \leftrightarrow \text{T}^\ulcorner \phi^\urcorner \wedge \text{T}^\ulcorner \psi^\urcorner).$$

It follows from this and Lemma 7.2 that the translation of KF9 under  $J$  is provable in  $\text{AT}^+$ . The axioms KF10–KF12 are dealt with in a similar way.

For the quantifier axioms KF13–KF16, the uniformity of the disquotation sentences is needed. Otherwise, these four axioms are verified much like KF9.

This concludes the proof of Theorem 5.2.

**§8. Some corollaries.** In this section, I outline some strengthenings, variants and consequences of the main result Theorem 5.2.

$\text{AT}^+$  is based on the uniform reflection scheme for the set  $\text{UDS}^+$  of type-free disquotation sentences. What happens if one allows *all* uniform T-positive disquotation sentences, not just those in which T does not occur, in the scope of a negation symbol? Obviously the resulting system is at least as strong as  $\text{AT}^+$ . Because of Lemma 6.2, it is not stronger, either. Thus no proper strengthening of  $\text{AT}^+$  can be achieved by allowing for occurrences of T in the scope of even numbers of occurrences of  $\neg$ .

$\text{AT}^+$  may also be weakened without any loss of proof-theoretic strength. The result of §7 still holds if the reflection axioms of  $\text{AT}^+$  are replaced by suitable reflection rules. An examination shows that the following two rules are sufficient for proving all (translations of) axioms of KF:

$$\frac{\forall \phi, \theta \in \mathcal{L}_T^+ \quad \text{Bew}_{\text{UDS}^+}(\ulcorner \psi(\ulcorner \phi^\urcorner, \ulcorner \theta^\urcorner)^\urcorner)}{\forall \phi, \theta \in \mathcal{L}_T^+ \quad \psi(\ulcorner \phi^\urcorner, \ulcorner \theta^\urcorner)} \quad \frac{\forall \phi(v) \in \mathcal{L}_T^+ \quad \text{Bew}_{\text{UDS}^+}(\ulcorner \psi(\ulcorner \phi(v)^\urcorner)^\urcorner)}{\forall \phi(v) \in \mathcal{L}_T^+ \quad \psi(\ulcorner \phi(v)^\urcorner)}$$

The resulting theory still interprets KF. The rules do not *imply* the reflection scheme as the uniform reflection rule for PA implies the uniform reflection scheme for PA (see Feferman [5, Theorem 2.19]). Feferman’s argument for reducing the axioms to the rule cannot be applied because  $\text{UDS}^+$  does not contain any arithmetical axioms.

The main result Theorem 5.2 still applies if  $\text{AT}^+$  is strengthened by formulating the reflection axiom, not only for  $\text{UDS}^+$ , but for finite subtheories of KF. This follows from Theorem 5.2 and the uniform reflexivity of KF (Lemma 6.4).

**COROLLARY 8.1.** *Let  $S$  be a finite subtheory of KF. Then PA with full induction and the reflection axioms*

$$\forall \vec{x} \quad (\text{Bew}_S(\ulcorner \phi(\vec{x})^\urcorner) \rightarrow \phi(\vec{x}))$$

*is  $\mathcal{L}_{\text{PA}}$ -conservatively interpretable in KF.*

Therefore one would not obtain a more versatile truth predicate by declaring the KF-axioms truth-analytic; the uniform T-disquotation sentences yield already the strength of  $RA_{<\epsilon_0}$ . Thus meaning postulates beyond the disquotational feature (i.e., the uniform disquotation sentences) are not necessary. Hence I consider Corollary 8.1 as a partial justification of the disquotationalist doctrine that truth serves its purpose in virtue of its disquotational feature, and that no meaning postulates going beyond the disquotational ones are needed.

The reflection principles of AT and AT<sup>+</sup> may be contrasted to other reflection principles in proof theory. In general, proof theorists have mostly investigated the result of adding the uniform reflection principle for  $S$  to a theory  $S$  itself. For instance, the result of strengthening PA by adding the uniform reflection principle for PA has been studied in detail. In contrast, UDS and UDS<sup>+</sup> are very weak theories compared to PA. The consistency of UDS<sup>+</sup> can be established in arithmetical theories much weaker than PA.

**§9. Open questions: Conservativeness.** What happens to AT<sup>+</sup> if the instances of the induction scheme are restricted to arithmetical formulas? The restricted theory is designated by AT<sup>+</sup>↓.

If induction in KF is restricted to arithmetical formulas only, the theory KF↓ is obtained. Cantini [1] showed that KF↓ is conservative over PA.

Thus one could try to interpret AT<sup>+</sup>↓ in KF↓  $\mathcal{L}_{PA}$ -conservatively, following the lines of §6; then the conservativeness of AT<sup>+</sup>↓ could be deduced from Cantini's result on KF↓. However, in order to show that AT<sup>+</sup> is a subtheory of KF, I made use of induction involving T in Lemma 6.4. Therefore the strategy fails.

On the model-theoretic side, there is an additional difference between KF↓ and AT<sup>+</sup>↓. Whereas every model of PA can be expanded to a model of KF↓, there are PA-models that cannot be expanded to models of AT<sup>+</sup>↓; for AT<sup>+</sup>↓ proves that there is a full satisfaction class, and thus every PA-model expandable to AT<sup>+</sup>↓ is recursively saturated by Lachlan's Theorem [22].

There remains also another open question with respect to conservativeness: what happens if AT (or AT<sup>+</sup>) is formulated with reflection for the theory of *local*, rather than uniform, disquotation sentences? Is the resulting theory conservative over PA? Some information on the truth-theoretic content of this system is yielded by the following result:

**DEFINITION 9.1.** The system ATL is the theory given by all PA-axioms with full induction and the following axiom of inference:

$$\forall \vec{x} (\text{Bew}_{\text{DS}}(\ulcorner \phi(\vec{x}) \urcorner) \rightarrow \phi(\vec{x})).$$

**PROPOSITION 9.2.** *The system ATL proves all uniform disquotation sentences, i.e., all sentences of UDS. Moreover, it proves that truth commutes with the connectives (but presumably not with the quantifiers) on arithmetical sentences.*

Whether ATL is conservative over PA remains an open problem.

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