(Semi)simple, simply conn., cpt Lie grp G

max torus T, pos. simpl roots $\{\alpha_i\}_i$

$\rightsquigarrow$ · classical r-mat $r = \sqrt{-1} \sum_{\alpha} d_\alpha (F_\alpha \otimes E_\alpha - E_\alpha \otimes F_\alpha) \in \mathfrak{g} \otimes \mathfrak{g}$

· Poisson structure on G $f_i \in \mathbb{C}[G] \subset U(\mathfrak{g})^*$

$$(\{f_0, f_1\}, T) = (f_0 \otimes f_1, [\hat{\Delta}(T), r])$$

Quantization $\quad \underbrace{(U(G_q), \hat{\Delta}_q}, R_q) = U_q(\mathfrak{g})$

$\qquad\qquad\qquad\qquad \updownarrow$ duality

$\qquad\qquad\qquad\qquad (\mathbb{C}(G_q), \Delta_q) = \mathbb{C}[G_q]$.

$h = -\log q$

· $\frac{1}{h}(R_q - 1) \to r \quad (h \to 0)$

· lin iso $f \to f^{(q)}$ $\mathbb{C}[G] \to \mathbb{C}[G_q]$ s.t.

$$\frac{1}{\sqrt{-1}\, h} [f_0^{(q)}, f_1^{(q)}] \to \{f_0, f_1\}.$$

"Twisting" from classification of cpt qgrp.

· $r \rightsquigarrow r + \sum \omega_{ij} H_i \otimes H_j$ $\quad ((\omega_{ij})_{i,j}$ antisym.
"2-cocycle" (Belavin-Drinfeld) on $\mathfrak{h} = \mathrm{Lie}\, T$.

$\leftrightsquigarrow$ Rieffel deformation for ω,
w.r.t. the adjoint action of T on G_q

· "3-cocycle" Φ : $U(1)$-valued 3-cocycle on $\widehat{ZG}$

→ twist the rep cat of G_q by

$\Phi(\pi_U|_{ZG}, \pi_V|_{ZG}, \pi_W|_{ZG}) : (U \otimes V) \otimes W \to U \otimes (V \otimes W)$

Now (C^*-) $\otimes$-category $(\mathrm{Rep}\, G_q, \Phi)$.

NY '13) $\exists$ q grp G_q^τ s.t. $\mathrm{Rep}\, G_q^\tau \simeq^{\otimes} (\mathrm{Rep}\, G_q, \Phi)$

$\tau \in Z(G)^{\mathrm{rk}\, G} \twoheadrightarrow H^3(Z\hat{G}; U(1))$

again $T < G_q^\tau$.

$\rightsquigarrow$ Rieffel deformation $G_q^{\tau,\omega}$

Q. which of them are isomorphic to each other?

Woronowicz's Tannaka-Krein duality

compact quantum grp G $\longleftrightarrow$ $\left\{\begin{array}{l}\circ\ C^*\text{-}\otimes\text{-cat w/ duality } \mathcal{C} = \mathrm{Rep}\, G \\ \circ \text{ tensor functor } \mathcal{C} \to \mathrm{Hilb}_f\end{array}\right.$

q-grp hom $H \to G$ $\longleftrightarrow$
$$\begin{array}{ccc}\mathrm{Rep}\, G & \to & \mathrm{Rep}\, H \\ & \searrow \ \ \swarrow & \\ & \mathrm{Hilb}_f & \end{array}$$

$\rightsquigarrow$ ∘ CQG's w/ same rep. category $\longleftrightarrow$ different fiber functors on the same $\otimes$-cat

∘ CQGs w/ same cat & dim $\longleftrightarrow$ 2-cocycles on discrete dual

∘ isomorphisms $\longleftrightarrow$ autoequivalences $\mathcal{E}: \mathcal{C} \to \mathcal{C}$
$F: \mathcal{C} \to \mathrm{Hilb}_f$ and $F\mathcal{E}$ def. isom q grps

"3-cocycle twisting is nontriv"

Prop G simple $(\mathrm{Rep}\, G_q, \Phi) \underset{\sim}{\overset{\otimes}{\to}} (\mathrm{Rep}\, G_q, \Phi')$

$\Rightarrow \Phi = \Phi'$ in $H^3(\widehat{ZG}; U(1))$

Idea : Pick X irred generating obj of $\mathrm{Rep}\, G_q$

"faithful rep of G_q"

Specify $f: 1 \hookrightarrow X^{\otimes n}$ only using fusion rule

$\rightsquigarrow X \simeq 1 \otimes X \overset{f \otimes 2}{\hookrightarrow} X^{\otimes n} \otimes X \xrightarrow[\text{using } \Phi]{\text{assoc.}} X \otimes X^{\otimes n} \xrightarrow{2 \otimes f^*} X \otimes 1 \simeq X$

is a sclar, reflecting the class of Φ

(other than D_{2n}, $H^3(\widehat{ZG}; U(1)) \simeq \mathbb{Z}/n \to U(1)$.

Φ root data of G.

- $\mathrm{Aut}(\Phi) \simeq \mathrm{Out}(G) = \mathrm{Aut}(G)/\mathrm{Inn}\, G$.

  $\wr$ McMullen

  $\mathrm{Aut}(\underbrace{R^+(G)}_{\text{rep semiring}})$ $\bigoplus_{U: \mathrm{Irrep}\, G} \mathbb{N}[U]$

- $\sigma \in \mathrm{Aut}(\Phi)$ induces aut of $U_q(\mathfrak{g})$ $E_i \mapsto E_{\sigma i}, K_i \mapsto K_{\sigma i}$

$\mathrm{Aut}^{\otimes}(\mathrm{Rep}\, G_q^{\tau}) \to \mathrm{Aut}(R^+ G_q^{\tau}) = \mathrm{Aut}(R^+ G) = \mathrm{Aut}(\Phi)$

NT '10) $\mathrm{Aut}^{\otimes}(\mathrm{Rep}\, G_q) \simeq H^2(\widehat{ZG}; U(1)) \rtimes \mathrm{Aut}\, \Phi$

Key : The kernel of $\mathrm{Aut}^{\otimes}(\mathrm{Rep}\, G_q) \to \mathrm{Aut}\, \Phi$ is :

- the autoeq. preserving isom classes.
- invar. 2-cohom $H^2_{G_q}(\hat{G}_q; U(1))$

$C^* ZG \to U(G_q)$ induces isom $H^2(\widehat{ZG}; U(1))$

$\simeq H^2(\hat{G}_q, U(1))$

how ?

1st strategy : define $c(\xi, \eta) : P_+ \times P_+ \to \mathbb{C}^\times$

from the action of $\mathcal{E}$ on $V_{\xi+\eta} \subset V_\xi \otimes V_\eta$.

analyze & show that c is indused by some

alt. bichar on ZG.

2nd strategy :
$$\begin{array}{ccc} \mathrm{Rep}\, G_q & \xrightarrow{\mathcal{E}} & \mathrm{Rep}\, G_q \\ \pi \downarrow & & \downarrow \pi \\ P(\mathrm{Rep}\, G_q; M) & \overset{\tilde{\mathcal{E}}}{\dashrightarrow} & P(\mathrm{Rep}\, G_q; M) \end{array}$$
P-bdary

Def $\mathrm{Aut}(\Phi)_\Phi$ = stab of Φ w.r.t. the induced action $\mathrm{Aut}\, \Phi \curvearrowright H^2(\widehat{ZG}; U(1))$

NY) $\underset{G \text{ simple}}{\exists}$ ext. $H^2(\widehat{ZG}; U(1)) \to \mathrm{Aut}^{\otimes}(\mathrm{Rep}\, G_q^\tau) \to \mathrm{Aut}(\Phi)_\Phi$.

Rem. G simple & not D_{2n} $\rightsquigarrow$ $\mathrm{Aut}\, \Phi \curvearrowright H^2(\widehat{ZG}; U(1))$ is triv.

Summary : the only isoms between $G_q^{\tau, \omega}$

come from

1) symmetry of root sys.

2) (antisym) bichar. on $\hat{ZG}$.

Sorting out cohomology bus;

Th'm type A case. $SU_q^{\tau,\omega}(n) \cong SU_q^{\tau',\omega'}(n)$ iff

either

1) $\omega_{ij}^2 \prod_{k=i}^{j-1} \tau_k = \omega_{ij}'^2 \prod_{k=i}^{j-1} \tau_k'$ for any $i<j$

2) $= \omega'_{n-i+1,n-j+1} \prod \bar{\tau}'_{n-k}$