

## EXERCISES 2

- ① Show that a linear map  $T: V \rightarrow W$  between normed spaces is bounded  $(\Leftrightarrow) T^{-1}(\{w \in W \mid \|w\|_W \leq 1\})$  has a non-empty interior.
- ② Let  $T: V \rightarrow W$  be a linear map between normed spaces. Why is the implication  $(T \text{ is bounded} \Rightarrow \text{Ker } T \text{ is closed})$  true and its converse is false?
- ③ Give an example of an unbounded linear map between normed spaces.
- ④ Show that all linear maps from a finite-dimensional normed space to a normed space are bounded.