Variable exponent theory and applications
Banach Center IM PAN, Warsaw

October 26-29, 2014

Abstracts

October 2, 2014

Sylwia Barnaś, Cracow University of Technology and **Iwona Skrzypczak**, University of Warsaw, Poland

Title: *Hardy inequality in variable exponent Lebesgue spaces derived from nonlinear problem*

Abstract: We derive a family of weighted $p(x)$–Hardy inequalities with an additional term of the form

$$\int_\Omega |\xi|^{p(x)}\mu_1(dx) \leq \int_\Omega |\nabla\xi|^{p(x)}\mu_2(dx) + \int_\Omega |\xi \log \xi|^{p(x)}\,\mu_3(dx),$$

where $1 < p^- := \operatorname{ess\,inf}_{x\in\Omega} p(x) \leq p(x) \leq p^+ := \operatorname{ess\,sup}_{x\in\Omega} p(x) < \infty$, $\xi : \Omega \to \mathbb{R}$ is compactly supported Lipschitz function, and Ω is an open subset of $\mathbb{R}^n$, not necessarily bounded. The involved measures $\mu_1(dx), \mu_2(dx)$ depend on $p(x)$, a certain parameter β, a piecewise continuous function $\sigma(x)$, and a nonnegative weak solution u to the PDI

$$-\Delta_{p(x)}u \geq \Phi \quad \text{in} \quad \Omega,$$

with a locally integrable function Φ. We admit the functions $\sigma(x)$ and Φ for which there exists

$$\sigma_0 := \inf_{x\in\Omega} \left\{\sigma(x) : \Phi \cdot u + \sigma(x)|\nabla u|^{p(x)} \geq 0\right\} \in \mathbb{R}.$$

As a consequence of Caccioppoli–type inequality for the solution u we get Hardy inequality with an additional term in variable exponent Lebesgue spaces.

Piotr Michał Bies, Warsaw University of Technology, Poland

Title: *Schauder theory in variable Hölder spaces*

Abstract: We study partial differential equations of second order with the right side in variable Hölder space. We show that solutions of this problem are in variable Hölder

space and that they are uniqueness. We are doing it by proving Schauder estimates in such spaces. The talk is based on results obtained together with P. Górka.

Verena Bögelein, Friedrich-Alexander University, Erlangen-Nürnberg, Germany

Title: *Parabolic problems with variable exponent growth*

Abstract: In this talk we are concerned with regularity properties of parabolic equations and systems with variable exponent growth. The model problem we have in mind is the parabolic $p(x,t)$-Laplacian

$$\partial_t u - \mathrm{div}(|Du|^{p(x,t)-2}Du) = 0.$$

In particular, we discuss the self-improving property of higher integrability, Hölder continuity of the spatial gradient Du of the solution and a Calderón–Zygmund theory. We will briefly outline the techniques used in the proof of these results.

Micha Gaczkowski, Warsaw University of Technology, Poland

Title: *Variable Sobolev spaces on Riemannian manifolds*

Abstract: In this talk we are going to introduce variable Sobolev space on Riemannian manifolds. Continuous and compact embedding will be discussed in the case of complete manifold. For non compact manifolds, compact embedding will require a space of functions invariant under the action of some group. As an application we will study the PDE problem involving $p(x)$-Laplacian. The talk is based on results obtained together with P.Górka.

Marek Galewski, Institute of Mathematics, Technical University of d, Poland

Title: *On a new multiple critical point theorem and some applications to anisotropic problems*

Abstract: Using the Fenchel-Young duality and mountain pass geometry we derive a new multiple critical point theorem. In a finite dimensional setting it becomes three critical point theorem while in an infinite dimensional case we obtain the existence of at least two solutions. The applications to anisotropic problems show that one can obtain easily that all critical points are nontrivial.

Leszek Gasiński, Jagiellonian University, Cracow, Poland

Title: *Positive Solutions for the Dirichlet $p(z)$-Laplacian with Concave and Convex Nonlinearities* (based on a joint work with Nikolaos S. Papageorgiou)

Abstract: We consider a nonlinear parametric Dirichlet problem driven by the anisotropic p-Laplacian with the combined effects of "concave" and "convex" terms:

$$\begin{cases} -\Delta_{p(z)}u(z) \ = \ \lambda u(z)^{q(z)-1} + f\big(z, u(z)\big) \text{ in } \Omega, \\ u|_{\partial\Omega} = 0, \ \lambda > 0. \end{cases}$$

The "superlinear" nonlinearity need not satisfy the Ambrosetti-Rabinowitz condition. Using variational methods based on the critical point theory and the Ekeland variational principle, we show that for small values of the parameter, the problem has at least two nontrivial smooth positive solutions.

Przemysław Górka, Warsaw University of Technology, Poland

Title: *On (pre)compactness in variable exponent spaces*

Abstract: We shall discuss the characterization of relatively compact subsets of the variable Lebesgue space on metric measure spaces. Moreover, compact embedding of variable Hajłasz-Sobolev spaces on compact metric spaces will be presented.

References

[1] M. Gaczkowski, P. Górka, *Variable Hajłasz-Sobolev spaces on compact metric spaces*, submitted.

[2] P. Górka, A. Macios, *Almost everything you need to know about relatively compact sets in variable Lebesgue spaces*, forthcoming.

Petteri Harjulehto, Oulu University, Finland

Title: *Harnacks inequality*

Abstract: Does weak solutions of the $p(x)$-Laplace equation satisfy Harnacks inequality? I give an overview to the topic.

Peter Hästö, Oulu University, Finland

Title: *The maximal operator on Musielak–Orlicz spaces*

Abstract: In this talk I present a sufficient condition for the boundedness of the maximal operator on Musielak–Orlicz spaces. The result includes as special cases the optimal condition for Orlicz spaces as well as the essentially optimal the log-Hölder condition for variable exponent Lebesgue spaces.

Niklas L.P. Lundström, Umeå University, Sweden

Title: *The boundary Harnack inequality for variable exponent p-Laplacian* (based on a joint work with Tomasz Adamowicz)

Abstract: We show the boundary Harnack inequality for $p(\cdot)$-harmonic functions for domains in $\mathbb{R}^n$ satisfying the ball condition ($C^{1,1}$-domains) assuming the variable exponent p is a bounded Lipschitz function. The proof relies on construction of barrier functions and chaining arguments.

Olli Toivanen, IMPAN Warsaw, Poland

Title: *The $p(\cdot)$-Laplace equation and its generalizations*

Abstract: I present some recent results in the study of the variable exponent $p(\cdot)$-Laplace equation and its generalizations into other problems involving non-standard growth.