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On the computational problems concerning the non-least squares approximation

Let consider the approximation being defined by the functional

$$J = \|f - y\|^p_{L^p[-1,1]} = \int_{-1}^{1} |f(x) - y(x)|^p \, dx = MIN, \quad 1 \leqslant p < \infty, \tag{1}$$

where

$$y(x) = \sum_{j=0}^{r} a_j \phi_j(x) \tag{2}$$

$$(\phi_j, \phi_k)_{L^2[-1,1]} = \delta_{k,j} \|\phi_k\| 2_{L^2[-1,1]} = \int_{-1}^{1} [\phi_k(x)]^2 \, dx \quad \text{(Legendre polynomials).} \tag{3}$$

Then the minimization of the functional (1):

$$\frac{\partial J}{\partial a_j}, \quad 0 \leqslant j \leqslant r, \tag{4}$$

leads us to a system of nonlinear equations for $p \neq 2$ very difficult to treat, especially for non-integer p.

We shall demonstrate that the problem stated above can be replaced by a least squares problem using the Gegenbauer polynomials (special cases are: for $p = 1$ Chebyshev polynomials of second kind, for $p = 2$ Legendre polynomials, $p = \infty$ Chebyshev polynomials of first kind), and even by discrete interpolation of function $f(x)$ on the set of appropriate Gauss–Gegenbauer set of nodes in the interval $[-1, 1]$, and we can return to the original orthogonal polynomial set too.

Another problem concerns the well known Remez iterative algorithm:

$$J_N = \int_{-1}^{1} [f(x) - y_n(x)]^2 |f(x) - y_{n-1}(x)|^{p-2} \, dx = MIN, \quad p > 2, \; n \in \mathbb{N}, \tag{5}$$

$$y_n(x) = \sum_{j=0}^{r} a_{j,n} \phi_j(x), \quad n \in \mathbb{N} \cup \{0\}, \tag{6}$$

$$y_0(x) = \sum_{j=0}^{r} a_{j,0} \phi_j(x) \implies a_{j,0} = \frac{(f, \phi_j)_{L^2[-1,1]}}{\|\phi_j\|_{L^2[-1,1]}}, \quad 0 \leqslant j \leqslant r, \tag{6a}$$

which is theoretically convergent to the solution of our problem, but in practice causes several problems especially when we operate with the $l^p[-1, 1]$ space instead of space $L^p[-1, 1]$, associated to the numerical artefacts occurring then.

Finally we can introduce the interpolatory constraints in our algorithm. For the discrete case of the Remez algorithm, the author designed the programs NJÖRD and NJÖRDFB (improved). Some numerical tests will be shown.

References

[1] W. Gautschi, *Orthogonal Polynomials, Algorithms and Applications*, Springer, Berlin, Heidelberg, New York 2004.

[2] W. Golde, C. Norek, S. Paszkowski, *An Outline of The Approximation Theory and its Applications in Electrotechnics* (in Polish), PWN Warsaw 1958.