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Retrial queueing systems with changeable service rate

We deal with retrial queueing systems of the type $E/E/1/N$, $N \leqslant \infty$, in which customers arrive in a Poisson process with rate λ. The retrial time distribution is exponential with parameter ν. The service time distribution function is an exponential for both primary and repeated calls and the parameter of that distribution is defined in the following way. If at the instant a customer (primary or repeated) gets to service we have j, $j \geqslant 1$ customers in the system, then the parameter of service time is equal to μ_j.

Let $\xi(t)$ denote the number of customers in the orbit at the time t. If at the time t the server is busy and rate of the service is μ_i, $i \geqslant 1$, then we say that the server is in the phase i. If at the time t the server is free we say that the server is in the phase 0. Let now $\eta(t) \in \{0, 1, 2, \dots\}$ denote the phase of the server at time t. The process $(\eta(t), \xi(t))$ is a homogeneous Markov process with state space $E = \{(i, j), i \geqslant 0, j \geqslant \max\{0, i-1\}\}$.

Theorem. *Let* $\overline{\lim}_{j\to\infty}\lambda/\mu_j < 1$. *Then for the process* $(\eta(t), \xi(t))$ *the ergodic distribution exists and can be presented in the form*

$$\pi_{00} = \Big(1 + \sum_{i=1}^{\infty} b(i)A_i\Big)^{-1}, \quad \pi_{0j} = \pi_{00}\frac{\lambda}{j\nu}\sum_{k=1}^{j}\frac{\beta_k^{j-k}A_k}{(k-1)!\mu_k}, \quad j \geqslant 1,$$

$$\pi_{ij} = \pi_{00}\frac{\beta_i^{j-i+1}A_i}{(i-1)!\mu_i}, \quad j \geqslant i-1,\ i \geqslant 1,$$

where $\varkappa = -\lambda/\nu$, $\beta_i = \lambda/(\lambda+\mu_i)$, $A_i = -\nu\sum_{k=1}^{i}\varkappa^k H(k, i-k)$,

$$b(i) = \frac{1}{(i-1)!\mu_i}\left[\frac{\lambda+\mu_i}{\mu_i} + \frac{\varkappa}{\beta_i^i}\Big(\ln(1-\beta_i) + \sum_{k=1}^{i-1}\frac{\beta_i^k}{k}\Big)\right]$$

and the function $H(i, k)$ *is defined in the process of proof.*

Some optimization problems for such systems are considered.

References

[1] G.I. Falin, J.G.C. Templeton, *Retrial queues*, Chapman and Hall, London 1997.