

Zygmunt Ratajczyk and Henryk Kotlarski

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Zygmunt Ratajczyk (1949–1994)

Born 2nd May 1949

1967–1972 studies at Warsaw University

1972 MSc under Professor Andrzej Mostowski

assistant of A. Mostowski at Warsaw University

1978 PhD under Professor Wiktor Marek

assistant professor (*adiunkt*) at Warsaw University

died 22nd February 1994

- problem of expandability of models of ZF to models of KM,
- the foundations of arithmetic.

At the end of his life he was also interested in computational complexity.

Axiomatizations of ZF^{KM} and ZF^{KM_n} , where KM_n is the fragment of KM with the axiom of class existence restricted to Σ_n^1 formulas only and $ZF^T = \{\varphi \in L(ZF) : T \vdash \varphi\}$ for $T \subseteq KM$.

He gave such axiomatizations in his doctoral dissertation using ramified analysis.

The problem of the number of possible KM-expansions of a given model of ZFC.

The characterization (in the language of infinite games) of expandability to models of KM and of expandability to non- β -models of KM.

$PA(S)$ is an extension of Peano arithmetic PA obtained by adding a binary predicate S and axioms stating that S is a full inductive satisfaction class

The problem of axiomatizability of the theory
 $PA^{PA(S)} = \{\varphi \in L(PA) : PA(S) \vdash \varphi\}$

Ratajczyk constructed a recursive set of sentences (a modification of Ramsey's theorem) Φ such that $\Delta_0\text{-}PA + \Phi$ is an axiomatization of $PA^{PA(S)}$ where $\Delta_0\text{-}PA$ is PA with induction restricted to bounded formulas only.

An example of a combinatorial sentence independent of $PA(S)$.

A combinatorial criterion for the existence of full inductive satisfaction classes.

The latter problem was studied by Ratajczyk together with H. Kotlarski.

A system of ω -logic was constructed and there were given necessary and sufficient conditions – in the language of consistency of this system or its fragments – for the existence of full inductive and full Σ_m -inductive satisfaction classes ($m \in \omega$) over countable recursively saturated models of PA.

New axiomatizations of the theories $\text{PA}^{\text{PA}(S)}$ and $\text{PA}^{\Sigma_m\text{-PA}(S)}$.

In the second paper a similar problem was considered but this time transfinite induction instead of ω -logic was used.

Fragments of arithmetic, more exactly $I\Sigma_n$ ($I^-\Sigma_n$), i.e. Σ_n induction with (without) parameters.

A combinatorial analysis of functions provably recursive in $I\Sigma_n$.

A simple proof of the estimation of the growth for those functions.

Theorem on the estimation of growth of functions provably total in $I^-\Sigma_n$ in the terms of Hardy's hierarchy

The last two papers of Ratajczyk devoted to arithmetic were “Arithmetical transfinite induction and hierarchies of functions” and “Subsystems of true arithmetic and hierarchies of functions”. They constituted his *Habilitationsschrift*.

They belong to the new domain called combinatorial proof theory. It grew up from works of J. Paris, L. Kirby, L. Harrington, R. Solovay and J. Ketonen on independent sentences and is connected with fast growing functions.



Helsinki, Airport, LC'90



Henryk Kotlarski (1949–2008)

Born 25th February 1949

1967–1972 studies at Warsaw University

1972 MSc under Professor Andrzej Mostowski

assistant at Institute of Mathematics of the Technical University in Wrocław and in Pedagogical School in Kielce

1976 PhD under Professor Czesław Ryll-Nardzewski at the University of Wrocław

since 1977 assistant professor (*adiunkt*) and since 1984 associate professor at Szkoła Główna Gospodarstwa Wiejskiego

1982 received *Habilitation*

1993–2000 at High Agriculture–Pedagogical School in Siedlce

1999 the title of professor awarded by the president of Poland

2000–2008 professor at University of Cardinal Stefan Wyszyński in
Warsaw

died 17th February 2008

Scientific works:

- well-ordered models (connected with PhD thesis),
- recursively saturated models of Peano arithmetic PA,
- nonstandard satisfaction classes,
- automorphisms of recursively saturated models of PA,
- generalizations and strengthenings of classical results of mathematical logic and the foundations of mathematics,
- combinatorics of α -large sets.

recursively saturated models of Peano arithmetic PA:

showed among others that every recursively saturated model of PA has infinitely many non-isomorphic elementary cuts.

nonstandard satisfaction classes:

every countable recursively saturated model of PA has a full satisfaction class

pathological properties

Δ_0 -inductive full satisfaction classes: description of models having such classes

Together with Ratajczyk: full inductive or full partially inductive classes in general, conditions for existence of such classes (formulated in the language of consistency of certain ω -logics or in terms of transfinite induction).

automorphisms of recursively saturated models of PA:

11 papers devoted to automorphisms (some of them together with others)

extendability of automorphisms, open maximal subgroups of the group of automorphisms, the structure of subgroups of the group of automorphisms, etc.

generalizations and strengthenings of classical results:

new proofs of the first and second incompleteness theorems

constructed explicitly a model for $PA + \text{“}PA \text{ is inconsistent”}$

new proofs of Rosser's incompleteness theorem, of a theorem on arithmetical hierarchy and of Tarski's undefinability theorem.

papers devoted to the **combinatorics of α -large** (in the sense of Hardy hierarchy) sets

inspired by the fundamental work by J. Ketonen and R. Solovay

A monograph devoted to problems of model theory of arithmetic, combinatorics and proof theory.

It was planned to be dedicated to the memory of Zygmunt Ratajczyk. Unfortunately he was unable to finish this work.



West Berlin, LC'89



Henryk Kotlarski



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