Full Satisfaction Classes in a General Setting (Tutorial)

Ali Enayat and Albert Visser

A *full satisfaction class* on an $\mathcal{L}$-structure $\mathcal{M}$ with sufficient coding apparatus decides the 'truth' of every $\mathcal{L}$-formula with parameters in $\mathcal{M}$, *including the nonstandard formulas in* $\mathcal{M}$, while obeying the usual recursive Tarski conditions for a satisfaction predicate. In this tutorial we present a robust technique for building a wide variety of full satisfaction classes using *model-theoretic* ideas, in the setting of a flexible notion of 'base theory' that encompasses base theories as weak as bounded arithmetic and as strong as Zermelo-Fraenkel set theory. Our model-theoretic construction is also shown to be implementable in the fragment WKL_0 of Second Order Arithmetic, which in turn implies that the conservativity of B + "S is a full satisfaction class" over B can be verified in Primitive Recursive Arithmetic for every r.e. base theory B. We also investigate interpretability issues connected to satisfaction classes. In particular, we show that B + "S is a full satisfaction class" is *interpretable* in B for all *inductive* base theories B, such as B = Peano arithmetic, or B = Zermelo-Fraenkel set theory.