# XLVIII Conference
# Mathematical Statistics

**December 5 – 9, 2022**
**Będlewo, Poland**

# Contents

## Part VI. Index

Part I

# Introduction

The Mathematical Statistics Conference has been held annually since 1973 (except 1981 and 1989). From December 5 to December 9, 2022, the 48th edition will take place at the Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences in Będlewo.

The aim of the conference is to bring together scientists who share a common interest in mathematical statistics and its applications. The main topics covered at the conference include:

- parametric and nonparametric estimation,
- testing statistical hypotheses,
- linear and generalized linear models,
- planning of experiments,
- time series analysis,
- multidimensional data analysis,
- asymptotic analysis,
- graphical models.

The special emphasis will be also paid on novel methods and algorithms used in applied mathematics and computational statistics. The conference is theoretical in nature, however, the solutions presented during the speeches can be applicable in many areas of modern science and technology, including genetics, biology, economics, agriculture and engineering.

## Committees and organizers

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- Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology
- Mathematical Institute, University of Wrocław
- Banach Center, Institute of Mathematics of the Polish Academy of Sciences, Warsaw

Part II

# Conference program

# Program

## Monday, 5 December 2022

**8:00–9:00 Breakfast**

9:20–9:30 Opening

9:30–10:30 *A. Panorska:* From butterflies to atmospheric rives: a journey with heavy tails, I

**10:30–11:00 Coffee break**

11:00–11:20 *M. Bogdan:* Polya Tree prior shrinkage for the estimation in high-dimensional Generalized Linear Models

11:20–11:40 *D. Młynarczyk:* Bayesian methods for radiation biodosimetry

11:40–12:00 *M. Janiszewska:* On estimation of block covariance matrix with linearly structured blocks

**12:00–12:30 Coffee break**

12:30–12:50 *A. Mieldzioc:* Structure identification and estimation linearly structured covariance matrix

12:50–13:10 *B. Majewski:* Spectral Density Estimation for Spectrally Correlated Processes

13:10–13:30 *M. Balcerek:* Fractional Brownian motion with random Hurst exponent

**13:30 Lunch**

15:30–16:30 *A. Panorska:* From butterflies to atmospheric rives: a journey with heavy tails, II

**16:30–17:00 Coffee break**

17:00–17:20 *A. Wyłomańska:* Discriminating Gaussian processes via quadratic form statistics

17:20–17:40 *J. Janczura:* Probabilistic forecasts for the dynamic, short-term, risk management strategies in electricity markets

17:40–18:00 *K. Burnecki:* Tempered fractionally integrated process with stable noise

**18:30–19:30 Dinner**

## Tuesday, 6 December 2022

**8:00–9:00 Breakfast**

9:30–10:30 *A. Jakubowski:* Estimation of high quantiles for maxima of stationary processes

**10:30–11:00 Coffee break**

11:00–11:20 *B. Kołodziejek:* Model selection in the space of Gaussian models invariant by symmetry
11:20–11:40 *M. Łazęcka:* Conditional independence testing based on Conditional Randomisation and Permutation
11:40–12:00 *K. Maraj-Zygmąt:* Empirical moments statistics for testing of the processes with finite moments distributions

**12:00–12:30 Coffee break**

12:30–12:50 *W. Żuławiński:* Periodic autoregressive models with additive noise: finite- and infinite-variance cases
12:50–13:10 *J. Leśkow:* Functional data analysis for nonstationary signals
13:10–13:30 *E. Gajecka-Mirek:* Functional Data Analysis for periodically correlated time series

**13:30 Lunch**

15:30–16:30 *T. Kozubowski:* Laplace probability distributions and generalizations: An excursion beyond normality, I

**16:30–17:00 Coffee break**

17:00–17:20 *A. Geras:* A novel probabilistic model for cell type identification in spatial transcriptomics and single-cell RNA sequencing data
17:20–17:40 *K. Grzesiak:* A novel semiparametric model for hydrogen deuterium exchange monitored by mass spectrometry data
17:40–18:00 *S. Piątek:* Quantile versions of inequality curves and inequality measures and their estimation

**18:30–19:30 Dinner**

**19:30 Meeting of the Statistics Commission of the Committee on Mathematics of the Polish Academy of Sciences**

## Wednesday, 7 December 2022

**8:00–9:00 Breakfast**

**9:30–13:30 Excursion**

**13:30 Lunch**

15:30–16:30 *T. Kozubowski:* Laplace probability distributions and generalizations: An excursion beyond normality, II

**16:30–17:00 Coffee break**

17:00–17:20 *P. Sulewski:* Easily Changeable Kurtosis Distribution
17:20–17:40 *K. Pączek:* Conditional variance ratio as an estimator of alfa-parameter for symmetric stable distribution
17:40–18:00 *Y. G. Yatracos:* Limitations of the Wasserstein MDE

**19:00 Conference dinner**

## Thursday, 8 December 2022

**8:00–9:00 Breakfast**

9:30–10:30 *W.K. Härdle:* Cryptos, NFTs, Digital Assets

**10:30–11:00 Coffee break**

11:00–11:20 *J. Gajda:* Fractional operators and machine learning
11:20–11:40 *Ł. Rajkowski:* Local Dependence Graphs for Discrete Time Processes
11:40–12:00 *K. Furmańczyk:* A construction of a graphical model

**12:00–12:30 Coffee break**

12:30–12:50 *K. Rudaś:* Logistic regression for uplift modeling
12:50–13:10 *P. Teisseyre:* Cost-constrained feature selection using an information-theoretic approach
13:10–13:30 *J. Mielniczuk:* One class classification approach to variational learning from Positive Unlabeled Data

**13:30 Lunch**

15:30–15:50 *M. Szymkowiak:* Reversed Aging Intensity Functions
15:50–16:10 *A. Goroncy:* Upper bounds on expectations of order statistics based on the IRFR distributions
16:10–16:30 *T. Rychlik:* Conditions for finiteness and bounds for moments of $k$th record values

**16:30–17:00 Coffee break**

17:00–17:20 *W. Rejchel:* High-dimensionality categorical data and model selection
17:20–17:40 *S. Nowakowski:* Model selection with high-dimensional categorical real world data
17:40–18:00 *P. Pokarowski:* Knee Index Criterion for Generalized Linear Model Selection

**18:30–19:30 Dinner**

## Friday, 9 December 2022

**8:00–9:00 Breakfast**

9:30–9:50 *P. Nowak:* Estimation in the Cox model with grouped lifetimes
9:50–10:10 *A. Markiewicz:* Estimation of linearly structured covariance matrix
10:10–10:30 *M. Samordak:* Graded version of the stochastic dispersive order

**10:30–11:00 Coffee break**

11:00–11:20 *P. Grzegorzewski:* A new test of independence for fuzzy data
11:20–11:40 *K. Filipiak:* Testing covariance matrix under T-distributed models
11:40–12:00 *M. Mokrzycka:* Maximum likelihood estimation of the covariance matrix with compound symmetry structure in multivariate T-distribution
12:00–12:20 *G. Wyłupek:* Verifying the validity of exponentiality
12:20–12:30 Closing

**12:30 Lunch**

Part III

# Invited speakers

**Wolfgang Karl Härdle (Humboldt-Universität zu Berlin, Germany)**
Wolfgang Härdle completed his Dr. rer. nat. in Mathematics at Heidelberg University and received his habilitation in Economics at Bonn University. He was the founder and Director of Collaborative Research Center "Quantification and Simulation of Economic Processes" (1994–2003), Director of "Economic Risk" (2005–2016) and also of C.A.S.E. (Center for Applied Statistics and Economics) (2001–2014). He is currently heading the Sino-German Graduate School on "High dimensional non-stationary time series analysis". He is the Ladislaus von Bortkiewicz Professor at Humboldt-Universität zu Berlin and Director of the BRC the joint Blockchain Research Centre with Zurich U and ASE, Bukarest. His current research focuses on modern machine learning techniques, smart data analytics and the crypto currency eco system. He has published more than 40 books and more than 350 papers in top statistical, econometrics and finance journals. He is highly cited, and among the top scientists in mathematics and economics. He has also professional experience in financial engineering, structured product design and credit risk analysis. He is the Editor in Chief of the Springer Journal Digital Finance.

**Adam Jakubowski (Nicolaus Copernicus University in Toruń, President of the Bernoulli Society)**
Adam Jakubowski completed his PhD in mathematics at the faculty of Mathematics, Physics and Chemistry of Nicolaus Copernicus University. He received habilitation in mathematics from the faculty of Mathematics, Informatics and Mechanics of University of Warsaw. In 1998 he received the title of Professor of Mathematics. He is the full professor at the faculty of Mathematics, Physics and Chemistry of the Nicolaus Copernicus University. In 1993-1999 he was the associate dean and in 1999-2005 the dean of this faculty. In 1984-1986 and 1993-2002 he was the head of the Department of Probability Theory and Statistics and from 2003 he is the head of the Department of Probability Theory and Stochastic Analysis. His main area of interest are the advanced limit theorems in the theory of probability and stochastic processes. Recently he also works on the applications of stochastic methods for the digital image processing. His achievements in these areas were acknowledged in 2009 by the prestigious Fellowship of the Institute of Mathematical Statistics and by entrusting him the role of the President of the Bernoulli Society (2021-2023).

**Tomasz Kozubowski (University of Nevada, Reno, USA)**
Following a graduate study of applied mathematics at the University of Warsaw, Poland, Tomasz J. Kozubowski received MS in Statistics from the University of Texas, El Paso, and Ph.D. in Statistics and Applied Probability from University of California, Santa Barbara. He is currently a Professor

in the Department of Mathematics & Statistics at the University of Nevada, Reno. Dr. Kozubowski works in the general area of stochastic modeling of natural phenomena in variety of fields, including climate research, geosciences, finance, and economics. His research interests include distribution theory, Laplace distribution and its generalizations, limit theory for random sums, heavy tailed distributions, extremes, mathematical statistics, financial and insurance mathematics, stochastic models for hydro-climatic phenomena, and fractal scaling processes. He has co-authored two monographs and over 100 research papers in probability and statistics. Dr. Kozubowski is currently an editorial board member of several academic journals and an active reviewer, having refereed for over 100 different academic journals. With the 2016 Sentinel of Science Reward, he was recognized by Publons as one of the top researchers contributing to the peer review in the field of mathematics.

**Anna Panorska (University of Nevada, Reno, USA)**
Anna Panorska works in the areas of probability theory and mathematical statistics, with focus on solving the problems that come from applications involving data with heavy tails. In particular, she works on the mathematical and statistical treatment of random sums and maxima involving heavy tails. Her recent research interests include studying extreme events in the stochastic processes used to model weather, water, and ecology including the impacts of climate change. Examples of those include flood, drought, deluge, and heat waves. She also works with epidemiological, engineering, and sports data and modeling. Her collaborators include hydrologists, biostatisticians, climate researchers, atmospheric scientists, engineers, ecologists, and medical researchers in addition to statisticians and mathematicians. She received funding from the US National Science Foundation, US Department of Defense, Nevada Department of Environmental Protection, NASA, and the EU Framework 7 program.

# Cryptos, NFTs, Digital Assets

**Wolfgang Karl Härdle**

Blockchain Research Center, Center for Applied Statistics and Economics,
Humboldt-Universität zu Berlin, Germany

## Abstract

Bitcoin (BTC) has attracted a plethora of investors and professional traders and becomes an almost inevitable asset class in today's financial markets. Deribit, the largest exchange for crypto options, offers European-typed inverse options, which target to BTC in USD but have payoff denominated in BTC. However, analytical insights to inverse options remain scarce. The dynamics of the underlying BTC is well described by a stochastic volatility model but in pricing inverse options one meets numerical difficulties in calibration and hedging. Financial analytics for the practically useful stochastic volatility with correlated jumps model is provided and comparison with simpler nested models both in in-sample and out-of-sample pricing is given [1].
The big bang of non-fungible tokens (NFTs) has caused the birth of a brand new era for digital art. NFTs, driven by blockchain and smart contracts, provide both artists and art collectors an unprecedented marketplace which is equipped with more security, flexibility, publicity, and freedom to monetize. To precisely depict the NFT art market and offer a gauge for market volatility, we construct the price index – Digital Art Index (DAI) – using a hedonic regression on the top 10 liquid NFT art collections and provide an empirical overview on this emerging market [2]. Due to the common impact of outlying phenomenon in the art markets, we propose two innovative alternative procedures – Huberization and DCS-t filtering to robustify the index.

## Keywords

Cryptocurrency, Non-fungible token, Blockchain technology, Stochastic volatility model, Hedonic regression.

## References

1. Teng, H.-W. and W.K. Härdle (2022). Financial Analytics of Inverse BTC Options in a Stochastic Volatility World. Available at SSRN: https://ssrn.com/abstract=4238213 or http://dx.doi.org/10.2139/ssrn.4238213
2. Lin, M.-B., B. Wang, F. Bocart, C. Hafner and W.K. Härdle (2022). 代 DAI Digital Art Index: A robust price index for heterogeneous digital assets. Available at SSRN: https://ssrn.com/abstract=4279412

# Estimation of high quantiles for maxima of stationary processes

**Adam Jakubowski**

Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University, Toruń

## Abstract

The estimation of high quantiles of partial maxima of a stationary time series remains a serious challenge. The dominating approach is based on analysis of few largest values (like in the well-known Hill estimator or pick over the threshold method).
In the present lecture we shall examine the block quantiles method in conjunction with the single sequence method (the latter described in [1]). The proposed estimators perform very well in simulations, behave more stable comparing to the standard techniques and cover models which were intractable so far.
This is **a joint work with Wojciech Rejchel, Igor Rodionov and Patryk Truszczyński**.

## Keywords

stochastic extremes, high quantiles, tail index, extremal index, phantom distribution function.

## References

1. Doukhan, P., Jakubowski, A. & Lang, G. (2015), Phantom distribution functions for some stationary sequences. *Extremes*, Vol. 18, 697–725.
2. Jakubowski, A., Rejchel, W. & Truszczyński, P. (2020+), Estimation of the tail index by the method of block quantiles (*in preparation*)
3. Jakubowski, A. & Rodionov, I. (2022+), Estimation of high quantiles of extrema of stationary stochastic processes (*in preparation*)

# Laplace probability distributions and generalizations: An excursion beyond normality

**Tomasz J. Kozubowski**

Department of Mathematics & Statistics, University of Nevada, USA

## Abstract

Asymmetric Laplace distributions, which naturally arise in connection with random summation and quantile regression, offer an attractive and flexible alternative to the classical Gaussian distribution in a variety of settings, where the assumptions of symmetry and thin tail are too restrictive. There has been a growing popularity of the Laplace-based models since the publication of [1], which may be due to their flexibility and fundamental properties, which include a sharp peak at the mode, heavier than Gaussian tails, existence of all moments, infinite divisibility, random stability, and approximation of geometric sums. Since the latter arise quite naturally, these distributions provide useful models in diverse areas, such as biology, economics, engineering, finance, geosciences, and physics. We review fundamental properties of these models, which give insight into their applicability in these areas, and discuss some extensions, including discrete models, circular models, and multivariate matrix-variate generalized Laplace distributions, among others.

This presentation is based on joint work with K. Podgórski and other collaborators.

## Keywords

Asymmetric distribution, Circular data, Clustering and classification, Discretization, Distribution theory, Financial data, Gamma process, Heavy tail, Infinite divisibility, Matrix variate distribution, Mixture of Gaussian distributions, Non-Gaussian distribution, Quantile regression, Time series, Pareto distribution, Power law, Random summation, Stochastic process, Subordination, Variance gamma distribution

## References

1. S. Kotz, T.J. Kozubowski, and K. Podgórski, The Laplace Distribution and Generalizations: A Revisit with Applications to Communications, Economics, Engineering and Finance, Birkhäuser, Boston (2001).

# From butterflies to atmospheric rives: a journey with heavy tails

**Anna K. Panorska[1], Tomasz J. Kozubowski[1], Marek Arendarczyk[2], Fares Qeadan[3], Alexander Gershunov[4], Alexander Weyant[4]**

[1] Department of Mathematics and Statistics, University of Nevada, Reno, USA
[2] Institute of Mathematics, University of Wroclaw, Poland
[3] Parkinson School of Health Sciences and Public Health, Loyola University in Chicago, USA
[4] Climate Division, SCRIPPS Institution of Oceanography, University of California San Diego, USA

## Abstract

The goal of this sequence of two talks is to motivate research in statistics driven by problems from many scientific disciplines. We present examples of current scientific questions solved using stochastic methods and models. While some questions are easier than others to solve, our work on the solutions led to development of new models and mathematical as well as statistical theory for those. The problems we present come from different research areas, but they have a common property: importance of the information found in the tails of distributions. In particular, we present research involving heavy tails. The parameters of the models we discuss tell a powerful and interesting story of the change of climate, and the subsequent change in the risk of disastrous extreme events or change in the populations of animals. One model we spend time on is $(X, Y, N)$, where $X$ describes the magnitude, $Y$ the maximum, and $N$ the duration of events such as drought, heat wave, cold spell, extreme wind, or deluge. The distribution of the vector $(X, Y, N)$ is of direct interest to water management, energy management companies, disaster management, health departments, environmental protection, as well as state and federal regulatory agencies. We present some very fresh results on the storms hitting the western US, with focus on the atmospheric rivers among other examples.

## Keywords

Stochastic models, heavy tails, extreme events, climate change

## Acknowledgements

This research is partially supported by the National Science Foundation (USA), ATM -0503722 and ATM-0231781, UNR VPRI Travel Award, RARE grant, California Climate Change Center, California Energy Commission, NOAA Office of Global Programs, Southwest Climate Science Center.

Part IV

# Participants' Abstracts

# Slash distributions, generalized convolutions, and extremes

**Marek Arendarczyk[1], Tomasz J. Kozubowski, Anna K. Panorska[2]**

[1]Mathematical Institute, University of Wrocław, Poland
[2]Department of Mathematics and Statistics, University of Nevada, Reno

## Abstract

An $\alpha$-slash distribution built upon a random variable $X$ is a heavy tailed distribution corresponding to $Y = X/U^{1/\alpha}$, where $U$ is standard uniform random variable, independent of $X$. We point out and explore a connection between $\alpha$-slash distributions, which are gaining popularity in statistical practice, and generalized convolutions, which come up in probability theory in connection with generalizations of the standard concept of convolution of probability measures. Our theoretical results are illustrated by several examples involving standard and novel probability distributions and extremes.

## Keywords

Extreme value theory, Generalized convolution, Heavy tails, Slash distribution

## References

1. Arendarczyk, M., Kozubowski, T. J., Panorska, A. K. Slash distributions, generalized convolutions, and extremes. Accepted for publication in *Annals of the Institute of Statistical Mathematics*.

# Fractional Brownian motion with random Hurst exponent

**Michał Balcerek**

Hugo Steinhaus Center, Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology

## Abstract

Fractional Brownian motion, a Gaussian non-Markovian self-similar process with stationary long-correlated increments, has been identified to give rise to the anomalous diffusion behavior in a great variety of physical systems. The correlation and diffusion properties of this random motion are fully characterized by its index of self-similarity or the Hurst exponent. However, recent single-particle tracking experiments in biological cells revealed highly complicated anomalous diffusion phenomena that cannot be attributed to a class of self-similar random processes. In the talk I will present the proposed process that preserves the properties of the fractional Brownian motion at a single trajectory level; however, the Hurst index randomly changes from trajectory to trajectory. I will provide a general mathematical framework for analytical, numerical, and statistical analysis of the fractional Brownian motion with the random Hurst exponent. The presentation is a summary of [1].

## Keywords

fractional Brownian motion, Hurst exponent, double stochasticity

## References

1. Balcerek, M., Burnecki, K., Thapa, S., Wyłomańska, A., and Chechkin, A. (2022). Fractional Brownian motion with random Hurst exponent: accelerating diffusion and persistence transitions. *Chaos 32*, 093-114.

# Polya Tree prior shrinkage for the estimation in high-dimensional Generalized Linear Models

**Daniel Yekutieli[1], Asaf Weinstein[2], Jonas Wallin[3], Małgorzata Bogdan[4]**

[1] Department of Statistics and Operations Research, Tel Aviv University
[2] Department of Statistics and Data Science, Hebrew University of Jerusalem
[3] Department of Statistics, Lund University
[4] Institute of Mathematics, University of Wroclaw

## Abstract

Pólya trees form a popular class of prior distributions used in Bayesian nonparametrics. So far, they have been used mainly for the nonparametric density estimation. In this talk we will present their novel application in the context of estimating parameters in the high dimensional Generalized Linear Models. We will justify our approach with some theoretical results within the framework of the Bayesian decision theory and present results of simulations in the context of high dimensional linear and logistic regression.

# Tempered fractionally integrated process with stable noise

**Farzad Sabzikar[1], Jinu Kabala[2], Krzysztof Burnecki[3]**

[1]Department of Statistics, Iowa State University, Ames, IA 50011, United States of America
[2]Department of Computer Science, Iowa State University, Ames, IA 50011, United States of America
[3]Hugo Steinhaus Center, Faculty of Pure and Applied Mathematics, Wroclaw University of Science and Technology

## Abstract

We present here the autoregressive tempered fractionally integrated moving average (ARTFIMA) process obtained by taking the tempered fractional difference operator of the non-Gaussian stable noise [1]. The tempering parameter makes the ARTFIMA process stationary for a wider range of the memory parameter values than for the classical autoregressive fractionally integrated moving average, and leads to semi-long range dependence and transient anomalous behavior. We investigate ARTFIMA dependence structure with stable noise and construct Whittle estimators [2].

## Keywords

ARTFIMA process, long-range dependence, Whittle estimator, anomalous diffusion.

## References

1. Kabala, J., Burnecki, K. and Sabzikar, F. (2021). Tempered linear and non-linear time series models and their application to heavy-tailed solar flare data. *Chaos 31*, 113-124.
2. Sabzikar, F., Kabala, J. and Burnecki, K. (2022). Tempered fractionally integrated process with stable noise as a transient anomalous diffusion model. *Journal of Physics A: Mathematical and Theoretical 55*, 1–27.

# Testing covariance matrix under T-distributed models

**Katarzyna Filipiak[1], Tõnu Kollo[2]**

[1]Institute of Mathematics, Poznań University of Technology, Poland
[2]Institute of Mathematical Statistics, University of Tartu, Estonia

## Abstract

In recent literature the hypotheses related to the covariance structures are tested by various test, including likelihood ratio and Rao score tests. However, in the majority of the results the normality of the data is assumed, and thus they cannot be applied for, e.g., heavy tailed distributions. Therefore, in this talk we present the likelihood ratio test for testing hypothesis about the covariance matrix being equal to a specific symmetric, positive definite matrix.

## Keywords

Covariance matrix, Likelihood ratio test, Maximum likelihood estimators, multivariate T-distribution.

# A construction of a graphical model

**Konrad Furmańczyk**

Institute of Information Technology, Warsaw University of Life Sciences (SGGW)

## Abstract

We present a nonparametric construction of a graphical model. Our construction shows an undirected graph model that represents conditional independence for general random variables defined by the conditional dependence coefficient (see [2]). Graph structure learning is proposed by two steps selection procedure using in last step Rothman et al. approach [6]. Proposed graph recovery structure has been evaluated on artificial and real datasets.

## Keywords

Nonparametric graphical model, conditional dependence, model selection

## References

1. Azadkia M. FOCI- R package (2020). https://cran.r-project.org/web/packages/FOCI/FOCI.pdf
2. Azadkia M and Chatterjee S. (2020). A simple measure of conditional dependence. Preprint. Available at https://arxiv.org/abs/1910.12327
3. Furmańczyk, K. (2022). A construction of a graphical model, Preprint.
4. Lauritzen, S. (1996). *Graphical Models. Clarendon Press.* Oxford
5. Maathuis, M., Drton, M., Lauritzen, S., Wainwright, M. (2020). *Handbook of Graphical Models* (1st ed). Chapman and Hall/CRC.
6. Rothman A.J, Levina E, Zhu, J. (2009). Generalized Thresholding of Large Covariance Matrices. *JASA 104*, 177–186.

# Fractional operators and machine learning

**Janusz Gajda**

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## Abstract

We will discuss the appearance of fractional operators in the context of data analysis with the use of machine learning methods. We discuss the concept of fractional (non-integer order) differentiation on real data of four datasets based on stock prices of main international stock indexes: WIG 20, S&P 500, DAX and Nikkei 225. For fractionally differenced series we use artificial neural networks (ANN) to build a predictive model. Our work is based on the paper [1].

## Keywords

fractional differentiation, machine learning

## References

1. Rafał Walasek and Janusz Gajda. Fractional differentiation and its use in machine learning. *International Journal of Advances in Engineering Sciences and Applied Mathematics*, 13(2-3):270–277, aug 2021.

# Functional Data Analysis for periodically correlated time series

**Elżbieta Gajecka-Mirek[1], Jacek Leśkow[1], Pierre-Lucas Dupond**

[1]Faculty of Computer Science and Telecommunications, Cracow University of Technology

## Abstract

The so-called functional approach is becoming popular in modern statistics. It assumes that observations are functions, not random values. If we are dealing with very large datasets, we can treat the time series as a set of random functions. In practice, often, we deal with data that is not only very numerous but also shows periodic behavior. In the presented results, the use of a functional autoregressive model (FAR) for modeling periodic time series will be proposed. Moreover the method of identifying the period will be presented. The periodic FAR method will be applied to periodic signals observed with high frequency which are energy price and bearing data.

## Keywords

FDA, periodically correlated time series, periodic functional autoregressive model.

## Acknowledgements

This research is supported by the grant for statutory research at the Cracow University of Technology.

## References

1. Horváth, L. and Kokoszka, P. (2012). *Inference for Functional Data with Applications*. Springer Series in Statistics.
2. Kidziński, L., Kokoszka, P. and Neda, M.J. (2018). Principal components analysis of periodically correlated time series. *Journal of Time Series Analysis, vol. 39, no 4*, 502–522.
3. Kokoszka, P. and Reimherr, M. (2013). Asymptotic normality of the principal components of functional time series. *Stochastic Processes and their Applications, vol. 123*, 1546 – 1562.
4. Leśkow, J. and Skupień, M. (2019). An application of functional data analysis to local damage detection. *Statistics in Transition, Vol. 20, no 1*, 131 – 151.

# A novel probabilistic model for cell type identification in spatial transcriptomics and single-cell RNA sequencing data

**Agnieszka Geras[1,2], Ewa Szczurek[2]**

[1] Warsaw University of Technology, Warsaw, Poland
[2] University of Warsaw, Warsaw, Poland

## Abstract

Various high-throughput technologies measuring gene expression, such as single cell RNA-seq and spatial transcriptomics were developed to study the role of different cell types in regulating the function of a tissue or an organ. Although single-cell RNA sequencing enables the measurement of gene expression at the resolution of single cells, it lacks information on cells' positions within the tissue [1]. However, recently developed spatial transcriptomics (ST) [2,3] technology enables to retain this information. ST consists of RNA-sequencing measurements in distinct spots containing an unknown number of mixed cells. The *mini bulk* nature of the spot measurements poses a challenge to decompose those cell-type mixtures.
In this study, we propose a novel hierarchical Bayesian probabilistic model and a marker-gene-driven computational method that will simultaneously perform the two tasks 1) assigning cell types in single cell data and 2) decomposing ST mixtures and a new statistical method to infer the model's parameters.

## Keywords

probabilistic graphical models, Markov chain Monte Carlo, transcriptomics

## Acknowledgements

This research is partially supported by the Polish National Science Centre PRELUDIUM grant no 2021/41/N/ST6/03619.

## References

1. A. Andersson et al. (2018). Single-cell and spatial transcriptomics enables probabilistic inference of cell type topography, *Communications Biology*
2. Vivien Marx (2021). Method of the Year: spatially resolved transcriptomics. *Nat Methods*
3. E. Berglund et al. (2018). Spatial maps of prostate cancer transcriptomes reveal an unexplored landscape of heterogeneity, *Nature Communications*

# Upper bounds on expectations of order statistics based on the IRFR distributions

**Agnieszka Goroncy[1], Tomasz Rychlik[2]**

[1]Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University in Toruń
[2]Institute of Mathematics, Polish Academy of Sciences

## Abstract

Danielak [1], and Goroncy and Rychlik [2] presented the sharp positive upper mean-variance bounds on the expectations of order statistics with relatively high ranks based on independent identically distributed random variables with the decreasing and increasing failure rates, respectively. In this paper we determine analogous evaluations in the case when the parent distributions have increasing reversed failure rates (IRFR).

## Keywords

Order statistics, expected value, optimal bound, increasing reversed failure rate.

## References

1. Danielak, K. (2003), Sharp upper mean-variance bounds for trimmed means from restricted families. *Statistics*, **27**, 305–324.
2. Goroncy, A., Rychlik, T. (2016), Evaluations of expectations of order statistics and spacings based on IFR distributions. *Metrika*, **79**, 635–657.

# A new test of independence for fuzzy data

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## Abstract

The question of whether the two features are independent arises in various theoretical studies and practical applications. Many parametric and nonparametric tests of bivariate independence can be found in the literature, like the chi-square test for contingency tables, tests based on Pearson's, Kendall's, and Spearman's correlation coefficients, etc. The problem of testing independence becomes much more difficult when the available data are imprecise, incomplete, or vague. Although fuzzy modeling provides appropriate tools for dealing with uncertain data, some limitations of fuzzy random variables inhibit the straightforward generalization of the classical tests of independence into the fuzzy framework.
In our presentation we consider a new method for testing independence with fuzzy data based on the concept of distance covariance and distance correlation [2]. We show that by tailoring the distance covariance to random fuzzy numbers we can construct a permutation test suitable for the verification of independence with fuzzy data (see [1]). Our test appears quite universal since it works well against various alternatives and types of dependence.

## Keywords

Fuzzy data, Fuzzy numbers, Permutation test, Random fuzzy number, Test of independence.

## References

1. Grzegorzewski P. (2020). Testing independence with fuzzy data. In: *D. Ciucci et al. (Eds.), Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2022), Proceedings of IPMU 2022, CCIS 1602, pp. 520?531, 2022.*
2. Székely, G.J., Rizzo, M.L. and Bakirov N.K. (2007). Measuring and testing dependence by correlation of distances. *Annals of Statistics, 35*, 2769–2794.

# A novel semiparametric model for hydrogen deuterium exchange monitored by mass spectrometry data

**Krystyna Grzesiak[1], Weronika Puchała[2], Michał Dadlez[2], Małgorzata Bogdan[1], Michał Burdukiewicz[3]**

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[3]Medical University of Białystok

## Abstract

The hydrogen-deuterium exchange monitored by mass spectrometry (HDX-MS) is one of the methods for studying the structure of proteins. In principle, the proteins are incubated in heavy water. During the incubation more exposed residues undergo the exchange of amide hydrogens to deuters much faster than others [1]. HDX-MS associates the speed of hydrogen-deuterium exchange with the regional rigidity of a protein. Such stability may be affected by the biological state [2] (e.g., presence of a ligand or lack thereof). Therefore, the changes in the protein's molecular structure caused by the biological state are inferred from the differences in the speed of hydrogen-deuterium exchange.

We propose a novel test based on a mixed semiparametric model and ridge regression that allows for the accurate identification of regions with significantly different exchange speeds at two biological states. To assess its performance, we have compared it with existing HDX-MS data analysis methods. The spline models are not as exposed to the impact of outliers as other model-based tests because of their local characteristics. As distinct from all existing methods they are capable of modeling the deuterium uptake well along the time taking into account its variability what makes them more sensitive in discriminating between different biological states.

We simulated data of 72 peptides at different biological states. The analysis of rejection rate in the pairwise testing procedure at the significance level 0.05 showed that the type I error (false positives) of semiparametric test does not surpass 0.06 while its power (true positives) is greater, when compared to other methods, even about 0.6 in the tough cases when the actual differences are small.

## Keywords

deuterium uptake, hydrogen-deuterium exchange, mass spectrometry, HDX-MS, spline regression

## Acknowledgements

This research was financed by the Foundation for Polish Science (grant TEAM TECH CORE FACILITY/2016-2/2 Mass Spectrometry of Biopharmaceuticals - improved methodologies for qualitative, quantitative and structural characterization of drugs, proteinaceous drug targets and diagnostic molecules given to Michał Dadlez) and supported by the National Science Centre (grant 2021/43/O/ST6/02805 Feature screening and model regularization for k-mer representations of biological sequences given to Małgorzata Bogdan).

## References

1. A. Berger and K. Linderstrom-Lang. Deuterium exchange of poly-dlalanine in aqueous solution. *Archives of Biochemistry and Biophysics*, 69:106-118, 1957. cited on p. 194.
2. G. R. Masson, J. E. Burke, N. G. Ahn, G. S. Anand, C. Borchers, S. Brier, G. M. Bou-Assaf, J. R. Engen, S. W. Englander, J. Faber, et al. Recommendations for performing, interpreting and reporting hydrogen deuterium exchange mass spectrometry (hdx-ms) experiments.*Nature Methods*, 16(7):595-602, 2019. cited on p. 194.

# Probabilistic forecasts for the dynamic, short-term, risk management strategies in electricity markets

**Joanna Janczura**

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## Abstract

The price risk related to trading in electricity markets has increased significantly in the recent years, due to the ongoing markets liberalization and the growing renewable energy sources production. In this paper we propose a short-term risk management strategy for an electricity supplier, that utilizes diversification of the markets for electricity trade. Based on the day-ahead probabilistic forecasts of electricity prices we calculate predictions of different risk and profit measures taking into account a possible split of the traded energy among markets. Strategies aiming at the risk minimization, profit maximization or finding optimal trade-off between risk and return are applied for the German and Polish electricity markets. The obtained results show that diversifying the markets at which electricity is traded leads to higher profits than trading on the day-ahead market and, at the same time, lower risk than associated with trading on the intraday or balancing market. In each of the considered cases, except for volatility as a risk measure for the German market, the goal of the strategy has been achieved. Implementation of the dynamic strategies has improved the outcomes in terms of risk or profit, compared to the static ones. [1].

## Keywords

Electricity market, Risk management, Short-term forecasting, ARX model

## Acknowledgements

The research was partially financed by NCN Sonata grant No. 2019/35/D/HS4/00369.

## References

1. Janczura, J. and E. Wójcik (2022). Dynamic short-term risk management strategies for the choice of electricity market based on probabilistic forecasts of profit and risk measures. The German and the Polish market case study, *Energy Economics* 110, 106015.

# On estimation of block covariance matrix with linearly structured blocks

**Malwina Janiszewska, Augustyn Markiewicz**

Department of Mathematical and Statistical Methods, Poznań University of Life Sciences, Poznań, Poland

## Abstract

This talk deals with estimation of block covariance matrix with linearly structured blocks. The general methodology of shrinkage estimation where the quadratic spaces of the structure play a crucial role, is presented. Main goal of this presentation is complete the characteristics of quadratic spaces with spaces of special block structure. The quasi shrinkage method is based on convex combination of the least squares estimator and appropriately chosen target matrix (cf. [1]). The choice of target matrix set affects the properties of estimators. The candidates of the target matrix sets from quadratic subspaces are studied. The resulting estimators are compared with respect to statistical and algebraic properties.

## Keywords

Covariance matrix, Block linear structure, Orthogonal projection, Quadratic subspace, Shrinkage method

## References

1. Janiszewska M, Markiewicz A, Mokrzycka M (2022). Quasi shrinkage estimation of block structured covariance matrix. - submitted

# Testing hypotheses related to covariance structures belonging to quadratic space under doubly multivariate model

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## Abstract

In this presentation hypothesis related to the covariance structure of doubly multivariate models are studied. For the hypothesis related to hierarchical dependence such that the interclass and intraclass correlations belong to the space of (commutative) quadratic structures the Rao score and likelihood ratio test statistics are derived as well as the exact distribution of the likelihood ratio test is determined.
Moreover, the hypothesis related to the independence of features between any two repeated measures in a block compound symmetry structure is tested. The distributions of likelihood ratio, Rao score and Wald tests are compared with respect to their convergence to the limiting distribution, as well as all of these tests are confronted with the F-test proposed by Fonseca et al., [1], and Roy's largest root test with respect to the power and robustness.
Presented results are applied to real data example.

## Keywords

Covariance structure, Quadratic space, Doubly multivariate model, Likelihood ratio test, Rao score test, Wald test, F-test, Roy's test

## References

1. Fonseca, M., A. Kozioł and R. Zmyślony (2018). Testing hypotheses of covariance structure in multivariate data, *Electron. J. Linear Algebra, 33* 53–62.

# Model selection in the space of Gaussian models invariant by symmetry

**Piotr Graczyk[1], Hideyuki Ishi[2], Bartosz Kołodziejek[3]**

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[2] Osaka City University
[3] Warsaw University of Technology

## Abstract

Colored Graphical Gaussian models (Graphical Gaussian Models with Edge and Vertex Symmetries) are especially useful when parsimony is needed. Apart from the conditional independence structure, symmetry restrictions are imposed on the concentration or partial correlation matrices. Such symmetries can be represented by a colored graph. Addition of symmetry to the conditional independence restrictions, reduces the number of parameters to estimate. Three types of such models (RCOP among them) were introduced in [3] to describe situations where some entries of concentration or partial correlation matrices are approximately equal. The RCOP models are simplest and most readily interpretable, partly due to their justification through permutation symmetries among the covariates and partly due to additional algebraic structure. In the talk I will describe Bayesian model selection procedure which applies to the case when conditional dependency graph $G$ is known and one looks for a graph coloring. The talk is based on [1] and [2].

## Keywords

Permutation symmetry, Colored Graphical model, Model selection.

## References

1. Graczyk, P., Ishi, H., Kołodziejek, B. and Massam, H. (2022) Model selection in the space of Gaussian models invariant by symmetry. *Ann. Statist.* 50, no. 3, 1747–1774.
2. Graczyk, P., Ishi, H. and Kołodziejek, B. (2022) Graphical Gaussian models associated to a homogeneous graph with permutation symmetries. arXiv:2207.13330
3. Højsgaard, S. and Lauritzen, S. L. (2008) Graphical Gaussian models with edge and vertex symmetries. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 70, no. 5, 1005–1027

# Functional data analysis for nonstationary signals

**Jacek Leśkow**

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## Abstract

In my presentation, I am going to show how methods of functional data analysis (FDA) can simplify modelling nonstationary signals when large data sets are under considerations. The talk will start with several examples and then the fundamental methods of FDA will be shown. At the end, several open problems will be presented.

## Keywords

Nonstationarity, functional data analysis, bootstrap

## Acknowledgements

Special thanks go to Dr Witold Cioch and Dr Paweł Pawlik for making the signals available for the analysis.

## References

1. Kokoszka, P. and Reimherr, M. (2017). *Introduction to Functional Data Analysis*, CRC Press.
2. Paparoditis, E. (2018). Sieve bootstrap for functional time series, *Ann. Statist. 46*, pp. 3510-3538.
3. Leśkow, J. and Skupień, M. (2019). An application of functional data analysis to local damage detecion, *Statistics in Transition 20*, pp. 131 - 151.

# Conditional independence testing based on Conditional Randomisation and Permutation

**Małgorzata Łazęcka[1,2], Bartosz Kołodziejek[1], Jan Mielniczuk[1,2]**

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## Abstract

We analyse properties of two resampling scenarios: Conditional Randomisation [1] and Conditional Permutation scheme which are relevant for conditional independence testing of discrete random variables $X$ and $Y$ given random variable $Z$.
First, we investigate asymptotic behaviour of estimates of a vector of probabilities based on both resampling scenarios, establish their asymptotic normality and ordering between asymptotic covariance matrices. Using these results we derive asymptotic distributions of empirical Conditional Mutual Information (Conditional Mutual Information is an information-theoretic measure of conditional dependence).
In numerical experiments we focus on small sample sizes. The simulations include the tests based asymptotic chi-square distribution with adjusted number of degrees of freedom [2], which is justified by the results mentioned above.

## Keywords

Conditional independence testing, Conditional Mutual Information, Conditional Randomisation, Conditional Permutation

## References

1. Candès, E., Fan, Y., Janson, L., Lv, J. (2018). Panning for gold: model-X knockoffs for high dimensional controlled variable selection. J. R. Stat. Soc., B: Stat. Methodol. 80 (pp. 551–577)
2. Tsamardinos, I., Borboudakis, G. (2010). Permutation testing improves Bayesian network learning. In: Lecture Notes in Computer Science vol. 6323 LNAI (pp. 322–337)

# Spectral Density Estimation for Spectrally Correlated Processes

**Anna Dudek[1], Bartosz Majewski[1], Antonio Napolitano[2]**

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## Abstract

For stationary processes, the support of the spectral measure is contained in the main diagonal in the bifrequency plane. In other words, spectral components of a stationary process at distinct frequencies are uncorrelated. However, if such a correlation exists, then it is an indicator of non-stationarity. An example is the class of spectrally correlated (SC) processes, i.e. harmonizable stochastic processes such that the support of their spectral measure is concentrated on countably many curves (support curves). In the special case where all support curves are lines with unit slopes, we obtain the important class of non-stationary processes, i.e. almost periodically correlated (APC) processes. Application of SC processes can be found in locating a moving source emitting a wide-band APC signal. Analysis of SC processes can be performed using the spectral densities located on the support curves. In our paper, we consider the case when support curves are unknown. In order to estimate spectral densities, a frequency-smoothed periodogram along the estimated support curve is proposed. For support lines with non-unit slopes, mean-square consistency is examined. Moreover, the estimation problem of the support line is discussed.

## Keywords

Estimated support curves, harmonizable processes, mean-square consistency, spectral density function.

## Acknowledgements

This work was partly supported by the King Abdullah University of Science and Technology (KAUST) Research Grant OSR-2019-CRG8-4057.2.

# Empirical moments statistics for testing of the processes with finite moments distributions

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## Abstract

In my presentation, I introduce a novel framework that allows efficient stochastic process discrimination. The underlying test statistic is based on even empirical moments [1] and generalises the time-averaged mean squared displacement framework. The test is designed to allow goodness-of-fit statistical testing of processes with stationary increments and finite moments distribution. In particular, while this test statistic is based on a simple and intuitive idea, it enables efficient discrimination between finite moments and infinite moments processes even if the underlying laws are relatively close to each other. This claim is illustrated via an extensive simulation study, e.g. where alpha-stable processes with stability index close to 2 are confronted with their standard Gaussian equivalents. For completeness, it is also shown how to embed this methodology into the real data analysis by studying the real metal price data.

## Keywords

goodness-of-fit test, even empirical moments, Gaussian process, alpha-stable process, Monte Carlo simulations, validation

## References

1. K. Maraj-Zygmąt and G. Sikora and M. Pitera and A. Wyłomańska. Goodness-of-fit test for stochastic processes using even empirical moment statistic. Submitted 2022.

# Estimation of linearly structured covariance matrix

**Augustyn Markiewicz**

Poznań University of Life Sciences

## Abstract

The paper deals with estimation of a linearly structured covariance matrix for multivariate and doubly multivariate data. It is known that the maximum likelihood estimators of covariance matrix have explicit representation if and only if the covariance structure space is a quadratic. We present the estimation method by projection of the maximum likelihood estimator of unstructured covariance onto the structure space.

## Keywords

Linear covariance structure, Quadratic space.

# Structure identification and estimation linearly structured covariance matrix

**Adam Mieldzioc**

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## Abstract

The paper aims to present the problem of identifying the structure of the covariance matrix using the entropy loss function, where the considered possible structures are linear.
For the selected structure, the positive definite and well-conditioned estimators are calculated using the shrinkage method with different target matrices. In simulation study, the correctness of the identification is checked and the statistical properties of the obtained estimators are compared.

## Keywords

Covariance matrix, Linear structure, Shrinkage, Entropy loss function

# One class classification approach to variational learning from Positive Unlabeled Data

**Jan Mielniczuk[1,2], Adam Wawrzeńczyk[1]**

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[2]Faculty of Mathematics and Information Sciences, Warsaw University of Technology, Warsaw

## Abstract

We present Empirical Risk Minimization approach in conjunction with variational inference method to learn classifiers for biased Positive Unlabeled (PU) data. For such a case, the labeling depends on covariates and thus the labeled observations are not a random sample from a positive class. We extend VAE-PU method introduced in [2] by proposing another estimator of theoretical risk of a classifier to be minimized, which has important advantages over the previous proposal. The modification is based on one class classification approach which turns out to be an effective method of detecting positive observations among unlabeled ones. Experiments performed on real data sets show that the proposed VAE-PU+OCC algorithm works very promisingly in comparison to its competitors such as the original VAE-PU and SAR-EM method [3] in terms of accuracy and F1 score.

## Keywords

positive and unlabeled data, empirical risk minimisation, variational auto-encoder, one-class classification

## References

1. Gong, C. et al. Instance-dependent positive and unlabeled learning with labeling bias estimation, IEEE Transactions on Pattern Analysis and Machine Intelligence, 2021
2. Na, B. et al., Deep generative positive-unlabeled learning under selection bias, CIKM 2020
3. Bekker et al., Beyond the SCAR assumption for learning from positive and unlabeled data, ECML 2019
4. Wawrzeńczyk, A. Jan Mielniczuk, One class classification approach to variational learning from biased positive unlabeled data, 2022, submitted

# Bayesian methods for radiation biodosimetry

**Dorota Młynarczyk[1], Pedro Puig[1,2], Carmen Armero[3], Virgilio Gómez-Rubio[4]**

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[3]Universitat de València,
[4]Universidad de Castilla-La Mancha

## Abstract

Biological dosimetry uses various statistical methods to estimate the radiation dose received by a person exposed to accidental or therapeutic radiation. The biological measure of the dose received is, among others, the frequency of certain biomarkers in the blood cells, which are used to determine the extent of radiation damage at the cellular level. It is therefore necessary to prepare dose-response calibration curves for particular biomarker, which are made by exposure of blood *in vitro* to controlled doses according to a certain study protocol. This enables for the establishment of some reference levels and, as a result, the estimation of the radiation dose.

One of the biomarkers of radiation exposure are chromosome translocations. It is well known that translocations can be found in blood cells many years after irradiation and thus are suitable for long-term evaluation. The well-established method says that the aberration yield is related to received dose by a linear quadratic equation. Then, for a given value, the frequency of translocations found in blood sample of a newly irradiated patient, an inverse method may be used to estimate the dose received by that individual. This study demonstrates some of the limitations of this approach, suggests future directions for its improvement, and provides justifications for the application of Bayesian inference.

Unlike chromosomal translocations, the $\gamma$-H2AX biomarker can only be used for up to 24 hours after irradiation due to its transient nature. The number of this phosphorylated protein in blood cells is highly dependent on the time elapsed after exposure and consequently this assay requires more sophisticated statistical techniques. We propose a model with two predictive variables: radiation dose and time since exposure, and instead of the calibration curve we consider a three-dimensional response surface. The Bayesian framework has the benefit in this scenario of being able to account for uncertainty in exposure time and hence generate more reliable results compared to the approach when the estimation is made at fixed post-irradiation times.

## Keywords

bayesian inference, biodosimetry, chromosome translocations, gamma-H2AX

## References


1. Abe, Y. *et al.* Dose-response curves for analyzing of dicentric chromosomes and chromosome translocations following doses of 1000 mGy or less, based on irradiated peripheral blood samples from five healthy individuals. *Journal of radiation research*, **59**(1), 35–42; doi.org/10.1093/jrr/rrx052 (2018).
2. López, J. S. *et al.* Establishment and validation of surface model for biodosimetry based on $\gamma$-h2ax foci detection.*International Journal of Radiation Biology*, **98**(1),1-10; doi.org/10.1080/09553002.2022.1998706 (2022).

# Maximum likelihood estimation of the covariance matrix with compound symmetry structure in multivariate T-distribution

**Katarzyna Filipiak[1], Daniel Klein[2], Stepan Mazur[3], Monika Mokrzycka[4]**

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## Abstract

In this talk we present the maximum likelihood estimators of the positive definite covariance matrix having compound symmetry structure under the multivariate T-distribution. For various sample sizes and degrees of freedom, the results will be compared with the corresponding estimators under the matrix normal distribution.

## Keywords

Covariance matrix, Compound symmetry structure, Maximum likelihood estimation, Multivariate T-distribution.

# Estimation in the Cox model with grouped lifetimes

**Piotr Nowak**

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## Abstract

In this talk we consider the problem of estimation in the Cox model when survival data are grouped. We show how random numbers can be used to transform grouped lifetimes into pseudo-complete sample. The Fisher consistency of partial likelihood estimator of the regression parameters in the Cox model based on the restored samples is investigated.
It is proved that for elliptical type distributional assumptions about explanatory variables the estimators of regression parameters in the Cox model based on the pseudo-complete sample are consistent up to a scaling factor.

## Keywords

survival data analysis, grouped data, Fisher consistency, elliptical distribution.

## References

1. Bednarski, T. (1993). Robust estimation in Cox's regression model. *Scandinavian Journal of Statistics,* 20, pp. 213-225.
2. Bednarski, T., Nowak, P. B. (2021). Scaled Fisher consistency of the partial likelihood estimator in the Cox model with arbitrary frailty. *Probability and Mathematical Statistics,* 41(1), pp. 77-87.

# Model selection with high-dimensional categorical real world data

**Szymon Nowakowski[1,2], Piotr Pokarowski[1], Wojciech Rejchel[3], Agnieszka Sołtys[1]**

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## Abstract

In this talk I will present `DMRnet` package written in `R`. This is a suit of algorithms dedicated to sparse model selection in high-dimensional categorical data settings. The algorithms involved have been undergoing heavy development recently both on the theoretical ground as well as on the implementation level. This practical (implementation) point of view will be the focus of my talk: I will show the results on real world data sets, for which a true model is not known, as well as on synthetic data generated from a known true model. I will show that the algorithms from the `DMRnet` package perform better than the state of the art algorithms with respect to the prediction accuracy or model dimension; with computational time shorter even by an order of magnitude.

## Keywords

model selection, high-dimensional data, categorical data

## Acknowledgements

This research is partially supported by the projects no. 501-D110-20-3004310 and 501-D111-20-0001370, contract no. 01/IDUB/2019/94.

# Conditional variance ratio as an estimator of $\alpha$-parameter for symmetric stable distribution

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## Abstract

One of the most issues in study behaviour of stable distribution are non defined higher moments and the lack of explicit formula for probability density function. Therefore the standard methods of estimation the parameters are not very effective or they require a long time of calculations.
We propose a new approach based on the conditional variance statistic [2] for estimating the stability index of symmetric stable distributions. During this talk we present the methodology and the simulation results. Moreover, the new approach will be compared with the well known methods for symmetric stable distribution [1].

## Keywords:

stable distribution, heavy-tailed distribution, conditional variance, estimation, tail index, stability index

## References

1. Nolan, J. P. (2020), *Univariate Stable Distributions. Models for Heavy Tailed Data*, Springer.
2. Pitera, M., Chechkin, A. and Wyłomańska, A. (2022), *Goodness-of-fit test for $\alpha$-stable distribution based on the quantile conditional variance statistics*, Statistical Methods & Applications 31, 387–424

# Quantile versions of inequality curves and inequality measures and their estimation

**Alicja Jokiel-Rokita, Sylwester Piątek**

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## Abstract

Classical inequality curves such as the Lorenz curve, the Bonferroni curve and the Zenga curves, as well as a new inequality curve (the $D$ curve) introduced in [1], are defined for distributions with finite mean value. Therefore, the inequality measures related to the above-mentioned inequality curves also apply when the expected value of an observable random variable is finite. Prendergast and Staudte [2,3] introduced three quantile versions of the Lorenz curve and a quantile version of the Zenga curve, and inequality measures based on these curves. We propose various nonparametric estimators of the quantile versions of the curves and indexes mentioned above, prove their consistency, and compare their accuracy in a simulation study. We also give an example of their application in real data analysis.

## Keywords

nonparametric estimation, consistency, Monte Carlo simulation

## References

[1] Davydov Y. and Greselin F. (2020). Comparisons between poorest and richest to measure inequalty. *Sociological Methods & Research*, 49(2):526–561.

[2] Prendergast L. A. and Staudte R. G. (2016). Quantile versions of the Lorenz curve. *Electronic Journal of Statistics*, 10:1896–1926.

[3] Prendergast L. A. and Staudte R. G. (2018). A simple and effective inequality measure. *The American Statistician*, 72(4):328–343.

# Knee Index Criterion for Generalized Linear Model Selection

**Piotr Pokarowski**

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## Abstract

The knee (elbow) rule is a graphical way to find the number of clusters, principal components, and the tradeoff between Type I and Type II errors. I present the adaptation of the knee criterion to model selection: choose the point where the increase in the adjustment coefficient is no longer worth the increase in the model dimension. I derive conditions for selection consistency and show in numerical experiments a comparison of the rule with other rules.

# Local Dependence Graphs for Discrete Time Processes

**Wojciech Niemiro[1,2], Łukasz Rajkowski[1]**

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## Abstract

Local dependence graphs for discrete time processes encapsulate information concerning the dependence relationships between the past of the multidimensional process and its present state and as such can represent feedback loops. They were considered e.g. in [1]. Despite the apparent simplicity of the discrete time setting, some natural questions relating the conditional (in)dependence statements in the process to separation properties of the underlying local dependence graph seem to be missing from the literature. We give graphical characteristics for two kinds of conditional independencies: those occurring in Markov chains under the stationary regime and independencies between the past of one subprocess and the future of another given the past of the third subprocess. We conjecture that the most of our results can be generalised to the case of continuous time processes (under a suitable definition of local dependencies graph), like Continuous Time Bayesian Networks.

## Keywords

Bayesian Networks, Causality, Local dependence graphs

## References

1. Michael Eichler and Vanessa Didelez. Causal reasoning in graphical time series models. In *23rd Annual Conference on Uncertainty in Artifical Intelligence*, 2007.

# High-dimensionality categorical data and model selection

**Szymon Nowakowski[1], Piotr Pokarowski[1], Wojciech Rejchel[2]**

[1] University of Warsaw
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## Abstract

Model selection with categorical data is challenging even for a moderate number of variables, because one parameter is roughly needed to encode one category or level. To solve this problem we propose the two-step procedure: first, we reduce dimensionality of a problem using the well-known Group Lasso. Then we choose the final model using an information criterion preceded by clustering levels of individual factors.
We investigate theoretical and practical properties of the algorithm in a sparse high-dimensional scenario, where a sample size might be significantly smaller than a number of all variables, but it is much larger than a number of active variables.

# Logistic regression for uplift modeling

**Magdalena Szypulska[1], Krzysztof Rudaś[1,2]**

[1]Faculty of Mathematics and Information Science, Warsaw University of Technology
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## Abstract

To evaluate efficiency of medical treatment or a marketing campaign we divide our population randomly into two groups: treatment (on which action is taken) and control (on which action is not taken) and calculate difference of effects predicted in these groups.
In our work we concentrate on classification uplift problem, where we try to predict scoring describing if given observation should be treated or not. In our work we present new regularization method for uplift logistic models, which give us natural interpolation between two basic approaches (double and uplift method) and also regularize vector of coefficients used to obtain appropriate scoring.

## Keywords

Uplift modeling, logistic regression, regularization.

# Conditions for finiteness and bounds for moments of $k$th record values

**Tomasz Rychlik**[1], **Magdalena Szymkowiak**[2]

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## Abstract

We consider the standard and $k$th record values arising in sequences of independent identically distributed continuous and positive random variables with finite expectations. We determine necessary and sufficient conditions on the type of record $k$, its number $n$ and moment order $r$ so that the $r$th moment of the $n$ value of $k$th record is finite for every parent distribution. Under the conditions we present the optimal upper bounds on these moments expressed in the scale units being the respective powers of the first population moment. The theoretical results are illustrated by some numerical evaluations.

## Keywords

independent identically distributed random variables, continuous life distribution, record value, moment, sharp bound.

## Acknowledgements

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# Graded version of the stochastic dispersive order

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## Abstract

The presentation is devoted to proposing construction of graded versions of stochastic orders, which are modifications of classical stochastic orders [1]. Decision-making processes inevitably involve in comparing possible alternatives. If the information required to determine a preference is uncertain, such tasks become nontrivial. To determine which quantity is greater ("better") we usually use stochastic orders which, unfortunately, are only partial orders. As a result, many objects (distributions) are incomparable. To avoid such inconvenience some modifications of the classical stochastic orders have been developed [1,2]. In particular, graded versions allow to quantify the degree in which the relation between random variables is satisfied.
In this presentation we introduce two variants of graded dispersive order of random variables. We discuss their properties and relations between the proposed constructions. Moreover, methods of approximation of the measure of dispersive domination are considered and applied in statistical inference problems.
New permutation test and bootstrap test for comparing the dispersion of two distributions are proposed. Besides the general construction of the tests, results of simulation studies on properties of both tests are examined. Finally, we compare our tests with some well know non-parametric tests.

## Keywords

Stochastic orders, Dispersive order, Permutation test, Bootstrap test

## References

1. Pérez-Fernández R., Baz J., Díaz I., Montes S., *On a graded version of stochastic dominance*, In: Proc. EUSFLAT 2021, pp. 494-500.
2. Pérez-Fernández R., Baz J., *On the role of the considered measure in a quantile-based graded version of stochastic dominance*, In: Proc. IPMU 2022, pp. 483–493.
3. Shaked M., Shanthikumar J.G., *Stochastic orders*, Springer, 2007.

# Easily Changeable Kurtosis Distribution

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## Abstract

The goal of this presentation is to introduce the easily changeable kurtosis (ECK) distribution. The uniform distribution appears as a special cases of the ECK distribution. The new distribution tends to the normal distribution. Properties of the ECK distribution such as PDF, CDF, modes, injection points, quantiles, moments, moment generating function, Moors' measure, moments of order statistics, random number generator and the Fisher Information Matrix are derived. The unknown parameters of the ECK distribution are estimated by the maximum likelihood method. The Shannon, Renyi and Tsallis entropies are calculated. Illustrative examples of applicability and exibility of the ECK distribution are given.

## Keywords

Normal distribution, Modeling kurtosis, Departure from normality, Monte Carlo method.

## References

1. Azzalini, A. (1985). A class of distributions which includes the normal ones. *Scandinavian Journal of Statistics, 2(12)*, 171–178.
2. Sulewski, P. (2019). Two-Piece Power Normal Distribution. *Communications in Statistics - Theory and Method, 50(11)*, 2619–2639.
3. Sulewski, P. (2020). Modifed Lilliefors goodness-of-fit test for normality. *Communications in Statistics - Simulation and Computation, 51(3)*, 1199–1219.
4. Torabi, H. and Grane, A. (1961). A test of normality based on the empirical distribution function. *SORT, 1(40)*, 55–88.
5. Tsallis, C. (1988). Possible generalizations of Boltzmann-Gibbs statistics. *Journal of Statistical Physics, (52)*, 479–487.

# Reversed Aging Intensity Functions

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## Abstract

The reversed aging intensity function is defined as the ratio of the instantaneous reversed hazard rate to the baseline value of the reversed hazard rate. It analyzes the aging property quantitatively, the higher the reversed aging intensity, the weaker the tendency of aging. In the paper [1], a family of generalized reversed aging intensity functions is introduced and studied.

## Keywords

Reliability, Hazard rate function, Reversed hazard rate function, Aging intensity function, Reversed aging intensity function.

## Acknowledgements

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## References

1. Buono, F., Longobardi, M., Szymkowiak, M. (2022). On Generalized Reversed Aging Intensity Functions. *Ricerche di matematica 71*, 85–108.
2. Finkelstein, M. S. (2002). On the Reversed Hazard Rate. *Reliability Engineering & System Safety 78*, 71–75.
3. Szymkowiak, M. (2018). Generalized Aging Intensity Functions. *Reliability Engineering & System Safety 178*, 198–208.

# Cost-constrained feature selection using an information-theoretic approach

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## Abstract

We consider a cost-constrained feature selection task [1,2] in which the goal is to select a subset of features allowing to predict the class variable (or many class variables) while satisfying a user-specific maximal admissible budget. Unlike in traditional feature selection problem, we assume that there are costs assigned to features. The cost of the feature is often associated with obtaining its value, for example in medical applications it can be the value extracted by a diagnostic test. We propose a novel cost-constrained feature selection method based on a sequential forward selection procedure that adds a new candidate feature in each step to the set of already-selected features by maximizing a certain score function. The score function contains two terms. The first term measures the informativeness of the candidate feature in the context of already-selected features and is based on lower bound of the conditional mutual information. The second term is a penalty for the cost of the candidate feature. The trade-off between feature relevance ant its cost is controlled by cost factor parameter. Its optimal value should depend on available budget as well as on the number of relevant features. We propose a data-driven method of finding the optimal value of the cost factor. The work is based on joint research with Tomasz Klonecki (Polish Academy of Sciences, Poland) and Prof. Jaesung Lee (Chung-Ang University, Republic of Korea).

## Keywords

feature selection, information theory, conditional mutual information

## References

1. Teisseyre, P. and Klonecki, T. (2021). Controlling Costs in Feature Selection: Information Theoretic Approach. *Proceedings of the International Conference on Computational Science ICCS'21*,
2. Klonecki, T. and Teisseyre, P. and Lee, J. (2022). Cost-constrained feature selection in multilabel classification using an information-theoretic approach. *manuscript*.

# Discriminating Gaussian processes via quadratic form statistics

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## Abstract

Gaussian processes are a powerful tool for modelling and predicting various numerical data. Hence, checking their quality of fit becomes a vital issue. In this paper, we introduce a testing methodology for general Gaussian processes based on a quadratic form statistic. We illustrate the methodology on three statistical tests recently introduced in the literature which are based on the sample autocovariance function, time average mean-squared displacement and detrended moving average statistics. We compare the usefulness of the tests by taking into consideration three very important Gaussian processes: the fractional Brownian motion, which is self-similar with stationary increments (sssi), scaled Brownian motion, which is self-similar with independent increments (ssii) and Ornstein–Uhlenbeck (OU) process, which is stationary. We show that the considered statistics' ability to distinguish between those Gaussian processes is high and we identify the best performing tests for different scenarios. We also find that there is no omnibus quadratic form test, however, the detrended moving average (DMA) test seems to be the first choice to distinguish between the same process with different parameters. We also show that the detrended moving average method outperforms the Cholesky method. Based on the previous findings we introduce a novel procedure of discriminating between Gaussian sssi, ssii and stationary processes. Finally, we illustrate the proposed procedure by applying it to real-world data, namely the daily EURUSD currency exchange rates and show that the data can be modelled by the OU process.

## References

1. Balcerek M., Burnecki K., Sikora G., Wyłomańska A. (2021). Discriminating Gaussian processes via quadratic form statistics, *Chaos 31, 063101.*

# Verifying the validity of exponentiality

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## Abstract

Let $X_1, \ldots, X_n$ be independent identically distributed random variables with the cumulative distribution function $G$. Set $F(x) = 1 - \exp(x), x \in [0, +\infty)$, and define the corresponding scale parametric family of the cdfs by $\mathcal{F} = \{F_\theta(\cdot) : F_\theta(\cdot) = F(\cdot/\theta)\}$ for some $\theta > 0$.
We test the composite goodness-of-fit null hypothesis

$$\mathscr{H}: G \in \mathcal{F} \qquad \text{against the general alternative} \qquad \mathscr{A}: G \notin \mathcal{F}.$$

The null hypothesis asserts that the observations come from an exponential distribution with unknown scale parameter $\theta$.
In the talk, we discuss selected significant solutions of the testing problem $(\mathscr{H}, \mathscr{A})$ and introduce a new one.

## References

1. Csörgő, S. (2002). Weighted correlation tests for scale families. *TEST*, 11, 219-248.
2. del Barrio, E., Cuesta-Albertos, J.A., Matrán, C., Rodríguez-Rodríguez, J. M. (1999). Tests of goodness of fit based on the $L_2$-Wasserstein distance. *Annals of Statistics*, 27, 1230-1239.
3. de Wet, T. (2002). Goodness-of-fit tests for location and scale families based on a weighted L2-Wasserstein distance measure. *TEST*, 11, 89-107.
4. de Wet, T., Venter, J. H. (1973). A goodness of fit test for a scale parameter family distributions. *South African Statistical Journal*, 7, 35-46.
5. Grzegorzewski, P., Wieczorkowski, R. (1999). Entropy-based goodness-of-fit test for exponentiality. *Communications in Statistics - Theory and Methods*, 28, 1183-1202.
6. Henze, N., Meintanis, S. G. (2005). Recent and classical tests for exponentiality: a partial review with comparisons. *Metrika*, 61, 29-45.
7. Kallenberg, W. C. M., Ledwina, T. (1997). Data-driven smooth tests when the hypothesis is composite. *Journal of the American Statistical Association*, 92, 1094-1104.
8. Wyłupek, G. (2022). Verifying the validity of exponentiality. *In preparation.*

# Limitations of the Wasserstein MDE

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## Abstract

For univariate data from a Black-Box with input $\theta$ and underlying, unknown model with heavy tails, we observe in simulations and confirm theoretically that the relative efficiency of the minimum Kolmogorov distance estimate, $\hat{\theta}_{MKD}$, with respect to the Minimum Wasserstein distance estimate, $\hat{\theta}_{MWD}$, converges to zero as the sample size, $n$, increases. The disturbing findings for $\hat{\theta}_{MWD}$ are due to the unboudedness of the Wasserstein distance and the heavy tails of the underlying model. These findings, along with existing drawbacks of the Wasserstein distance for multivariate data, do not support its use in statistical inference, in particular for Black-Box models.

## Keywords

Black-Box data; Data Science; Efficiency; Kolmogorov distance; Minimum Distance Estimation; Wasserstein distance

## References

1. Yatracos, Y. G. (2022) Limitations of the Wasserstein MDE for univariate data. *Statistics and Computing* **32**, 95, https://doi.org/10.1007/s11222-022-10146-7.*(Open access)*

# Periodic autoregressive models with additive noise: finite- and infinite-variance cases

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## Abstract

Periodic autoregressive (PAR) time series is considered as one of the most common models of second-order cyclostationary processes. However, in the real applications, the signals with periodic characteristics may be disturbed by additional noise related to measurement device disturbances or to other external sources. The known estimation techniques dedicated for PAR models assume noise-free model, thus may be inefficient for such cases. Here, we propose several estimation techniques for the noise-corrupted PAR models, considering both finite- and infinite-variance distributions of innovations and additive noise. These methods are based on the Yule-Walker methodology, i.e., they utilize the autocovariance function in the finite-variance case and alternative measures of dependence in the infinite-variance case (e.g., covariation and fractional lower order covariance for $\alpha$-stable distribution). The efficiency of proposed techniques is presented in the Monte Carlo simulation study.

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Part V

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