

Pricing Vulnerable Options Using Good Deal Bounds

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Vulnerable Options

- Vulnerable options = options where the writer of the option may default, mainly trading on OTC markets
- BIS, the OTC equity-linked option gross market value in the first half of 2006 USD 6.8 tln

Previous Literature

- Treatment in complete markets (Hull-White(1995), Jarrow-Turnbull(1995), Klein(1996));
- Hung-Liu (2005) : market incompleteness and good deal bound pricing for vulnerable options. Only Wiener process setup.

Contributions of the current paper

- Streamlining the existing literature on vulnerable options in complete markets;
- Applying the Bjork-Slinko (2005) method of computing good deal bounds to obtain higher tractability;
- Applying structural methods for default (intensity based method - work in progress);
- Extending the results for european calls to options with homogeneous payoff functions of the first degree (e.g. exchange options) ;

Pricing in incomplete markets

pricing in incomplete markets \rightarrow no unique EMM
 \rightarrow no unique price

- classical solutions:
 - no-arbitrage bounds - *too large*
 - choosing one specific martingale measure
 - *ad-hoc; economic meaning?*
- alternative solution - GOOD DEAL BOUNDS
 - Cochrane and Saa Raquejo (2000)
 - Bjork and Slinko (2005)

Theory of Good Deal Bounds

Main Idea

set a bound on the possible Sharpe Ratio of any portfolio that can be formed on the market \leftrightarrow

\leftrightarrow set a bound on the possible Girsanov kernels for potential EMM

\leftrightarrow set a bound on the possible prices for the claim

Structural Model

specified under the **objective measure \mathbb{P}**

- traded stock S

$$dS_t = \alpha_t S_t dt + S_t \bar{\gamma}_t d\tilde{W}_t;$$

- assets of the counterparty Y

$$dY_t = \mu_t Y_t dt + Y_t \bar{\sigma}_t d\tilde{W}_t;$$

- bank account B ;
- the payoff function of the vulnerable option

$$\Phi(S_T, Y_T) = \max(S_T - K, 0)I(Y_T \geq D) + \mathcal{R}I(Y_T < D);$$

- recovery payoff

$$\mathcal{R} = (1 - \beta) \frac{Y_T}{D} \max[S_T - K, 0]$$

Good Deal Bound Problem

The **upper good deal bound** price process for a vulnerable option is defined the optimal value process for the following optimal control problem:

$$\max_{\varphi} E^Q[e^{-r(T-t)}(\max[S_T - K, 0]I\{Y_T \geq D\} + \mathcal{R}I\{Y_T \leq D\})]$$

$$dY_t = (\mu_t + \bar{\sigma}_t \varphi_t) Y_t dt + Y_t \bar{\sigma}_t dW_t$$

$$dS_t = rS_t dt + S_t \bar{\gamma}_t dW_t$$

$$\alpha_t + \bar{\gamma}_t \varphi_t = r$$

$$\|\varphi_t\|^2 \leq C^2$$

Hamilton Jacobi Bellman equation

The HJB equation:

$$\frac{\partial V}{\partial t}(t, s, y) + \sup_{\varphi} \mathcal{A}V(t, s, y) - rV(t, s, y) = 0$$
$$V(T, s, y) = \Phi(s, y).$$

is solved in 2 steps:

- solving for each t, s, y the embedded static problem
→ we obtain the Girsanov Kernel
- solving the PDE
→ we obtain the price of the vulnerable option

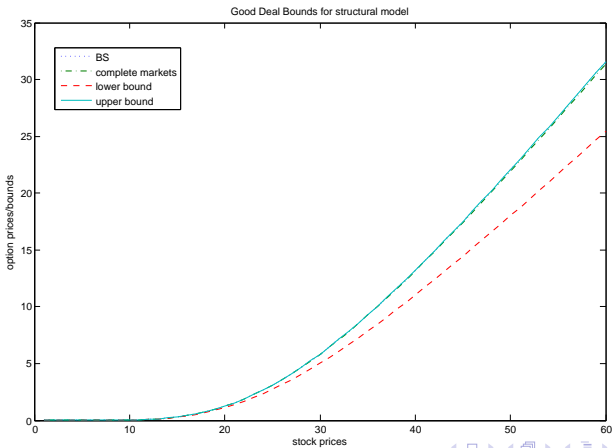
Results

- closed form solution for a vulnerable European call;
- closed form solution for other vanilla instruments, with payoff functions omogenous of the 1st degree in S , e.g exchange option;
- numerical solution for the case of a vulnerable option in the case of an untraded stock

Results for Vulnerable European Call

- closed form solution for a vulnerable European call

$$\begin{aligned}
 \Pi(t) &= S_t \mathcal{N}[-a_1, -b_1, \rho] \\
 &- e^{-r(T-t)} K \mathcal{N}[-a_2, -b_2, \rho] \\
 &+ \frac{1-\beta}{D} S_t Y_t \exp\left\{\int_t^T [\mu_s + \bar{\sigma}_s \hat{\varphi}_s + \bar{\sigma}_s \bar{\gamma}'_s] ds\right\} \mathcal{N}[-a_3; b_3; -\rho] \\
 &- e^{-r(T-t)} \frac{K(1-\beta)}{D} Y_t \exp\left\{\int_t^T (\mu_s + \bar{\sigma}_s \hat{\varphi}_s) ds\right\} \mathcal{N}(-a_4, -b_4, \rho)
 \end{aligned}$$



Factors that influence the size of the GDB interval

- factors that influence the size of the good deal bounds interval
 - distance to default
 - size of the good deal bound constraint (B)
 - volatility of the assets of the counterparty
 - correlation between the assets of the counterparty and the underlying
 - the size of the market price of risk for the underlying
 - the deadweight costs β

Extensions - exchange options

The payoff of a exchange option

$$\Phi(S_T^1, S_T^2, Y_T, T) = \max[S_T^1 - S_T^2, 0]I\{Y_T \geq D\} + \mathcal{R}I(Y_T < D) \quad (1)$$

- in complete markets, we can price an exchange option by change of measure
- can we apply the same techniques with GDB?

As in the complete market case:

- having a change of variable for the payoff and martingale conditions;
- re-stating the good deal bound condition:

$$\|\phi\|^2 \leq C^2 \rightarrow \|\psi - \bar{\gamma}'_2\|^2 \leq C^2 \quad (2)$$

- calculating the new relevant Girsanov kernel and correlation coefficient;
- substituting them in the formula for a European call

Extensions - Barrier options

In complete markets, we can remove the path dependency for some barrier options; results extend to vulnerable barrier options.

- can we extend results to GDB?
- only for the case when the assets of the counterparty and the underlying are independent

$$\begin{aligned}\Pi(0, \Psi_{LO}^V) &= e^{-rT} E_{0,s,y}^Q \left[\Psi_L^V(S_T, Y_T) \right] \\ &- e^{-rT} \left(\frac{L}{s} \right)^{\frac{2\tilde{r}}{\gamma^2}} E_{0, \frac{L^2}{s}, y}^Q \left[\Psi_L^V(S_T, Y_T) \right]\end{aligned}\quad (3)$$

How?

- We introduce a new process Z_t with the same dynamics as S_t , but starting point $\frac{L^2}{s}$
- Notice that, in this set-up, the payoff of any defaultable claim can be written as:

$$\Psi^V(S_T, Y_T) = \Psi(S_T)F(Y_T),$$

where $F(Y_T) = I\{Y_T \geq D\} + \frac{(1-\beta)Y_T}{D}I\{Y_T < D\}$.

- We can set up a new GDB problem with payoff function $\Psi_L^V(S_T, Z_T, Y_T)$ and solve for the Girsanov kernel as before

Conclusion

- We apply the GDB method to vulnerable options;
- We allow for structural models of default;
- We extend the results for European call vulnerable options to other vanilla options with payoff functions homogeneous of the first degree in S .
- We extend results for barrier options when the assets of the counterparty and the underlying are independent