Valuation of exotic, interest rate and credit derivatives in Lévy models

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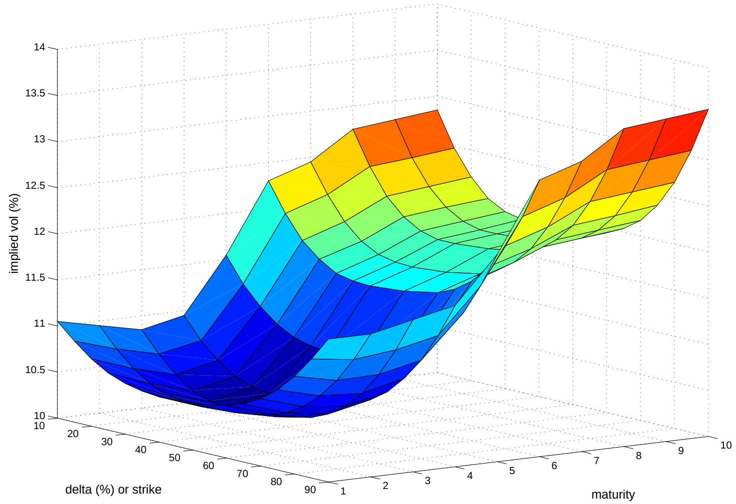
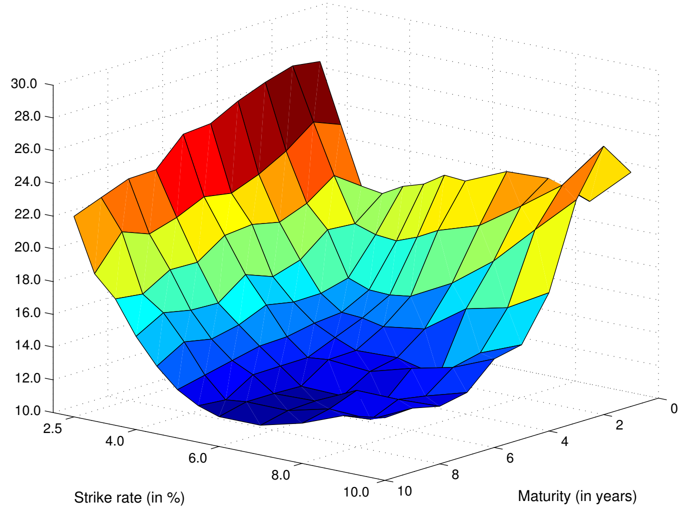
Volatility smile and surface





Volatility surfaces of foreign exchange and interest rate options

- Volatilities vary in strike (*smile*)
- Volatilities vary in time to maturity (*term structure*)
- Volatility clustering

Exponential semimartingale model

Let $\mathcal{B}_T = (\Omega, \mathcal{F}, \mathbf{F}, P)$ be a stochastic basis, where $\mathcal{F} = \mathcal{F}_T$ and $\mathbf{F} = (\mathcal{F}_t)_{0 \le t \le T}$. We model the price process of a financial asset as an exponential semimartingale

$$S_t = \mathrm{e}^{H_t}, \quad 0 \le t \le T. \tag{1}$$

$H = (H_t)_{0 \le t \le T}$ is a semimartingale with canonical representation

$$H = H_0 + B + H^c + h(x) * (\mu^H - \nu) + (x - h(x)) * \mu^H. \tag{2}$$

For the processes B, $C = \langle H^c \rangle$, and the measure ν we use the notation

$$\mathbb{T}(H|P) = (B, C, \nu)$$

which is called the *triplet of predictable characteristics* of the semimartingale H.

Martingale modeling

Let $\mathcal{M}_{\mathrm{loc}}(P)$ be the class of local martingales.

Assumption ($\mathbb{ES}$). The process $1_{\{x>1\}}\mathrm{e}^x * \nu$ has bounded variation.

Then

$$S = \mathrm{e}^H \in \mathcal{M}_{\mathrm{loc}}(P) \Leftrightarrow B + \frac{C}{2} + (\mathrm{e}^x - 1 - h(x)) * \nu = 0. \tag{3}$$

Throughout, we assume that P is a (local) martingale measure for S.

By the *Fundamental Theorem of Asset Pricing*, the value of an option on S equals the *discounted expected payoff* under a martingale measure.

We assume *zero* interest rates.

Supremum and infimum processes

Let $X = (X_t)_{0\le t\le T}$ be a stochastic process. We denote by $\overline{X} = (\overline{X}_t)_{0\le t\le T}$ and $\underline{X} = (\underline{X}_t)_{0\le t\le T}$, where

$$\overline{X}_t = \sup_{0\le u\le t} X_u \quad \text{and} \quad \underline{X}_t = \inf_{0\le u\le t} X_u$$

the supremum and infimum process of X respectively. Since the exponential function is monotone and increasing

$$\overline{S}_T = \sup_{0\le t\le T} S_t = \sup_{0\le t\le T} \left(\mathrm{e}^{H_t}\right) = \mathrm{e}^{\sup_{0\le t\le T} H_t} = \mathrm{e}^{\overline{H}_T}. \tag{4}$$

Similarly

$$\underline{S}_T = \mathrm{e}^{\underline{H}_T}. \tag{5}$$

Valuation formulae – payoff functional

We want to price an option with payoff $f(X_T)$, where $f(X_T) = f(H_t, 0 \leq t \leq T)$ is an $\mathcal{F}_T$-measurable functional.

The functionals we consider are "European style", and consist of two parts:

1. The *payoff function* is an arbitrary function $f : \mathbb{R} \to \mathbb{R}_+$; for example $f(x) = (\mathrm{e}^x - K)^+$ or $f(x) = 1_{\{\mathrm{e}^x > B\}}$, for $K, B \in \mathbb{R}_+$.

2. The *underlying process* can be the asset price or the supremum/infimum or an average of the asset price process (e.g. $X = H$ or $X = \overline{H}$).

- Exotic options

Valuation formulae – conditions

Conditions:

(R1) Assume that $\int_{\mathbb{R}} \mathrm{e}^{-Rx} f(x) \mathrm{d}x < \infty$ for all $R \in I_1 \subset \mathbb{R}$.

(R2) Assume that $M_{X_T}(z) = E[\mathrm{e}^{zX_T}] < \infty$, for all $z \in I_2 \subset \mathbb{R}$.

(R3) Assume that $I_1 \cap I_2 \neq \emptyset$.

Valuation formulae based on Fourier transforms; similar to Raible (2000), but *no* need for Lebesgue density.

Consider the Fourier transform of the *payoff function* like Borovkov and Novikov (2002); also Hubalek et al. (2006) and Černý (2007), for hedging.

Carr and Madan (1999) and Raible (2000) transform the *option price*.

Valuation formulae

Theorem 1. *Assume that conditions (R1)–(R3) are in force. Then, the price* $\mathbb{V}_f(X)$ *of an option on* $S = (S_t)_{0 \leq t \leq T}$ *with payoff function* $f = f(X_T)$ *is given by*

$$\mathbb{V}_f(X) = \frac{1}{2\pi} \int_{\mathbb{R}} \varphi_{X_T}(-u - iR) \mathfrak{F}_f(u + iR) \mathrm{d}u, \tag{6}$$

where φ_{X_T} *denotes the extended characteristic function of* X_T *and* $\mathfrak{F}_f$ *denotes the Fourier transform of* f.

Proof. Introduce the *dampened payoff function* $g(x) = \mathrm{e}^{-Rx} f(x)$, $R \in I_1$. Then

$$\mathbb{V}_f(X) = E[f(X_T)] = E[\mathrm{e}^{RX_T} g(X_T)] = \int_{\mathbb{R}} \mathrm{e}^{Rx} g(x) P_{X_T}(\mathrm{d}x). \tag{7}$$

Under assumption (R1), g has a Fourier transform $\mathfrak{F}_g$; inverting it, we get a representation as

$$g(x) = \frac{1}{2\pi} \int_{\mathbb{R}} \mathrm{e}^{-ixu} \mathfrak{F}_g(u) \mathrm{d}u. \tag{8}$$

Returning to the valuation problem (7) we get

$$\begin{aligned} \mathbb{V}_f(X) &= \int_{\mathbb{R}} \mathrm{e}^{Rx} \left(\frac{1}{2\pi} \int_{\mathbb{R}} \mathrm{e}^{-ixu} \mathfrak{F}_g(u) \mathrm{d}u \right) P_{X_T}(\mathrm{d}x) \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \mathrm{e}^{i(-u-iR)x} P_{X_T}(\mathrm{d}x) \right) \mathfrak{F}_g(u) \mathrm{d}u \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \varphi_{X_T}(-u - iR) \mathfrak{F}_f(u + iR) \mathrm{d}u. \end{aligned} \tag{9}$$

□

Valuation formulae II – options

Valuation formulae for options that depend on two functionals of the driving process.

Examples: *barrier*, *corridor* and *two-asset correlation* option

$$(S_T - K)^+ 1_{\{\overline{S}_T > B\}};$$

$$(S_T - K)^+ \sum_{i=1}^{N} 1_{\{L < S_{T_i} < H\}};$$

$$(S_T^1 - K)^+ 1_{\{S_T^2 > B\}}.$$

Valuation formulae II

Theorem 2. *The price* $\mathbb{V}_{f,g}(X,Y)$ *of an option on* $S=(S_t)_{0\le t\le T}$ *with payoff function* $f(X_T)g(Y_T)$ *is given by*

$$\mathbb{V}_{f,g}(X,Y)=\frac{1}{4\pi^2}\int\limits_{\mathbb{R}}\int\limits_{\mathbb{R}}\varphi_{X_T,Y_T}(-u-iR_1,-v-iR_2)$$
$$\times\,\mathfrak{F}_g(v+iR_2)\mathfrak{F}_f(u+iR_1)\mathrm{d}v\mathrm{d}u, \qquad (10)$$

where φ_{X_T,Y_T} *denotes the extended characteristic function of the random vector* (X_T,Y_T).

Proof. Conditions and proof are similar to Theorem 1. □

Examples of payoff functions

Example 3 (Call and put option)**.** Call payoff $f(x) = (\mathrm{e}^x - K)^+$, $K \in \mathbb{R}_+$,

$$\mathfrak{F}_f(u + iR) = \frac{K^{1+iu-R}}{(iu - R)(1 + iu - R)}, \qquad R \in I_1 = (1, \infty). \tag{11}$$

Similarly, if $f(x) = (K - \mathrm{e}^x)^+$, $K \in \mathbb{R}_+$,

$$\mathfrak{F}_f(u + iR) = \frac{K^{1+iu-R}}{(iu - R)(1 + iu - R)}, \qquad R \in I_1 = (-\infty, 0). \tag{12}$$

Example 4 (Digital option)**.** Call payoff $1_{\{\mathrm{e}^x > \underline{B}\}}$, $\underline{B} \in \mathbb{R}_+$.

$$\mathfrak{F}_f(u + iR) = -\underline{B}^{iu-R} \frac{1}{iu - R}, \qquad R \in I_1 = (0, \infty). \tag{13}$$

Similarly, for the payoff $f(x) = 1_{\{e^x < \overline{B}\}}$, $\overline{B} \in \mathbb{R}_+$,

$$\mathfrak{F}_f(u + iR) = \overline{B}^{iu-R} \frac{1}{iu - R}, \qquad R \in I_1 = (-\infty, 0). \tag{14}$$

Example 5 (Double digital option)**.** The payoff of a double digital call option is $1_{\{\underline{B} < e^x < \overline{B}\}}$, $\underline{B}, \overline{B} \in \mathbb{R}_+$.

$$\mathfrak{F}_f(u + iR) = \frac{1}{iu - R} \left(\overline{B}^{iu-R} - \underline{B}^{iu-R} \right), \qquad R \in I_1 = \mathbb{R} \backslash \{0\}. \tag{15}$$

Similar formulae for power options, self-quanto options, etc.

Lévy processes

Let $L = (L_t)_{0 \le t \le T}$ be a *Lévy process* with triplet of local characteristics (b, c, λ).

Assumption ($\mathbb{EM}$). There exists a constant $M > 1$ such that

$$\int_{\{|x|>1\}} \mathrm{e}^{ux} \lambda(\mathrm{d}x) < \infty, \qquad \forall u \in [-M, M].$$

Using ($\mathbb{EM}$) and Theorems 25.3 and 25.17 in Sato (1999), we get that

$$E\left[\mathrm{e}^{uL_t}\right] < \infty, \quad E\left[\mathrm{e}^{u\overline{L}_t}\right] < \infty \quad \text{and} \quad E\left[\mathrm{e}^{u\underline{L}_t}\right] < \infty$$

for all $u \in [-M, M]$.

Fluctuation theory for Lévy processes

Theorem 6 (Wiener–Hopf factorization). *Let* L *be a Lévy process. The Laplace transform of* $\overline{L}$ *at an independent and exponentially distributed time* θ *can be identified from the* Wiener–Hopf factorization *of* L *via*

$$E\left[\mathrm{e}^{-\beta\overline{L}_\theta}\right] = \frac{\kappa(q,0)}{\kappa(q,\beta)} \tag{16}$$

where $\kappa(\alpha,\beta)$, $\alpha \geq 0, \beta \geq 0$, *is given by*

$$\kappa(\alpha,\beta) = k \exp\left(\int_0^\infty \int_0^\infty (e^{-t} - e^{-\alpha t - \beta x})\frac{1}{t} P_{L_t}(\mathrm{d}x)\mathrm{d}t\right). \tag{17}$$

Moreover, κ *can be analytically extended to* $\alpha,\beta \in \mathbb{C}$ *with* $\Re a \geq 0$ *and* $\Re\beta \geq -M$.

Proof. Theorem 6.16 in Kyprianou (2006). □

Linking fixed and exponential times

Lemma 7. *Let* $L = (L_t)_{0 \le t \le T}$ *be a Lévy process that satisfies assumption* $(\mathbb{EM})$ *and consider* $\beta \in \mathbb{C}$ *with* $\Re\beta \in [-M, \infty)$*. The Laplace transforms of* $\overline{L}_t$*,* $t \in [0, T]$ *and* $\overline{L}_\theta$*,* $\theta \sim \mathrm{Exp}(q)$*, are related via*

$$E\big[\mathrm{e}^{-\beta \overline{L}_\theta}\big] = q \int_0^\infty \mathrm{e}^{-qt} E\big[\mathrm{e}^{-\beta \overline{L}_t}\big] \mathrm{d}t. \tag{18}$$

Moreover, the Laplace transform of $\overline{L}_\theta$ *is finite for* $\beta \in \mathbb{C}$ *with* $\Re\beta \in [-M, \infty)$*.*

Proof. An application of Fubini's theorem yields

$$E\big[\mathrm{e}^{-\beta \overline{L}_\theta}\big] = E\Big[\int_0^\infty q \mathrm{e}^{-qt} \mathrm{e}^{-\beta \overline{L}_t} \mathrm{d}t\Big] = q \int_0^\infty \mathrm{e}^{-qt} E\big[\mathrm{e}^{-\beta \overline{L}_t}\big] \mathrm{d}t.$$

□

On the characteristic function of the supremum I

Lemma 8. *Let* $L = (L_t)_{0\le t\le T}$ *be a Lévy process that satisfies assumption* $(\mathbb{EM})$. *Then, the moment generating function of* $\overline{L}_t$ *is defined for all* $u \in (-\infty, M]$ *and* $t \in [0,T]$.

Lemma 9. *Let* $L = (L_t)_{0\le t\le T}$ *be a Lévy process that satisfies assumption* $(\mathbb{EM})$. *Then, the characteristic function* $\varphi_{\overline{L}_t}$ *of* $\overline{L}_t$ *is holomorphic in the half plane* $\{z \in \mathbb{C} : -M < \Im z < \infty\}$ *and can be represented as a Fourier integral in the complex domain*

$$\varphi_{\overline{L}_t}(z) = E\left[\mathrm{e}^{iz\overline{L}_t}\right] = \int_{\mathbb{R}} \mathrm{e}^{izx} P_{\overline{L}_t}(\mathrm{d}x).$$

Lemma 10. *The map* $t \mapsto E\left[\mathrm{e}^{-\beta\overline{L}_t}\right]$ *is continuous for all* $\beta \in \mathbb{C}$ *with* $\Re\beta \in [-M, \infty)$.

On the characteristic function of the supremum II

Theorem 11. *Let $L = (L_t)_{0 \le t \le T}$ be a Lévy process. The Laplace transform of $\overline{L}_t$ at a fixed time t, $t \in [0, T]$, is given by*

$$E\left[\mathrm{e}^{-\beta \overline{L}_t}\right] = \frac{1}{2\pi} \int_{\mathbb{R}} \frac{\mathrm{e}^{t(Y+iv)}}{Y+iv} \frac{\kappa(Y+iv, 0)}{\kappa(Y+iv, \beta)} \mathrm{d}v, \tag{19}$$

for $Y > 0$. Moreover, the Laplace transform can be extended to the complex plane for $\beta \in \mathbb{C}$ with $\Re\beta \in [-M, \infty)$.

Proof. Combining eqs. (16) and (18) we get

$$q \int_0^\infty \mathrm{e}^{-qt} E[\mathrm{e}^{-\beta \overline{L}_t}] \mathrm{d}t = \frac{\kappa(q, 0)}{\kappa(q, \beta)}. \tag{20}$$

Applying Doetsch (1950), we invert the Laplace transform and the claim follows. □

Lookback options

Fixed strike lookback option: $(\overline{S}_T - K)^+$.
Combining Theorem 1, Theorem 11 and Example 3, we get

$$\mathbb{C}_T(\overline{S};K) = \frac{1}{2\pi}\int_{\mathbb{R}} \varphi_{\overline{L}_T}(-u-iR)\frac{K^{1+iu-R}}{(iu-R)(1+iu-R)}\mathrm{d}u \tag{21}$$

where

$$\varphi_{\overline{L}_T}(-u-iR) = \frac{1}{2\pi}\int_{\mathbb{R}} \frac{\mathrm{e}^{T(Y+iv)}}{Y+iv}\frac{\kappa(Y+iv,0)}{\kappa(Y+iv,u-iR)}\mathrm{d}v. \tag{22}$$

- The floating strike lookback option, $(\overline{S}_T - S_T)^+$, is treated by a *duality* formula.

One-touch options

One-touch call option: $1_{\{\overline{S}_T > B\}}$.
Combining Theorem 1, Theorem 11 and Example 4, we get

$$\mathbb{DC}_T(\overline{S}; B) = \frac{1}{4\pi^2} \int_{\mathbb{R}} \int_{\mathbb{R}} \frac{\mathrm{e}^{T(Y+iv)}}{Y+iv} \frac{\kappa(Y+iv,0)}{\kappa(Y+iv,iu-R)} \frac{B^{iu-R}}{R-iu} \mathrm{d}v\mathrm{d}u. \tag{23}$$

Similarly for the one-touch put option: $1_{\{\underline{S}_T \leq B\}}$.

$$\mathbb{DP}_T(\underline{S}; B) = \frac{1}{4\pi^2} \int_{\mathbb{R}} \int_{\mathbb{R}} \frac{\mathrm{e}^{T(Y+iv)}}{Y+iv} \frac{\widehat{\kappa}(Y+iv,0)}{\widehat{\kappa}(Y+iv,iu-R)} \frac{B^{iu-R}}{iu-R} \mathrm{d}v\mathrm{d}u. \tag{24}$$

Equity default swap (EDS)

- Fixed premium exchanged for payment at "default"
- default: drop of stock price by 30% or 50% of $S_0 \to$ first passage time
- fixed leg pays premium $\mathcal{K}$ at times $T_1, \ldots, T_N$, if $T_i \leq \tau_B$
- if $\tau_B \leq T$: protection payment, paid at time τ_B
- premium of the EDS chosen such that initial value equals 0; hence

$$\mathcal{K} = \frac{E\left[\mathrm{e}^{-r\tau_B} 1_{\{\tau_B \leq T\}}\right]}{\sum_{i=1}^{N} E\left[\mathrm{e}^{-rT_i} 1_{\{\tau_B > T_i\}}\right]}. \tag{25}$$

- Calculations similar to touch options, since $1_{\{\tau_B \leq T\}} = 1_{\{\underline{S}_T \leq B\}}$.

The Lévy forward rate model

The driving process $L = (L_t)_{0 \le t \le T}$ is a *time-inhomogeneous Lévy process*:

$$\mathbb{E}\left[e^{iuL_t}\right] = \exp \int_0^t \Big(iub_s - \frac{u^2 c_s}{2} + \int_{\mathbb{R}} (e^{iux} - 1 - iux)\lambda_s(\mathrm{d}x) \Big) \mathrm{d}s. \tag{26}$$

Dynamics of instantaneous forward rates:

$$f(t,T) = f(0,T) + \int_0^t \partial_2 A(s,T)\mathrm{d}s - \int_0^t \partial_2 \Sigma(s,T)\mathrm{d}L_s, \tag{27}$$

where Σ is a deterministic volatility structure, bounded; $0 \le \Sigma(s,T) \le M, \forall s \in [0,T]$.

Martingale condition: $A(s,T) = \theta_s(\Sigma(s,T))$; θ_s cumulant associated to (b_s, c_s, λ_s).

Money market account $B_t^M := \exp(\int_0^t r_s \mathrm{d}s)$ and prices of zero coupon bonds:

$$B_T^M = \frac{1}{B(0,T)} \exp\Big(\int_0^T A(s,T)\mathrm{d}s - \int_0^T \Sigma(s,T)\mathrm{d}L_s \Big), \tag{28}$$

and

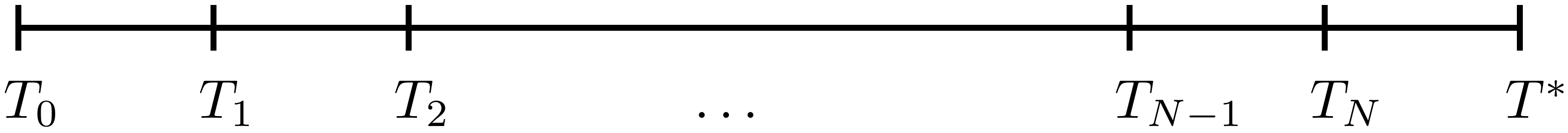
$$B(T,U) = \frac{B(0,U)}{B(0,T)} \exp\Big(- \int_0^T A(s,T,U)\mathrm{d}s + \int_0^T \Sigma(s,T,U)\mathrm{d}L_s \Big), \tag{29}$$

where: $\Sigma(s,T,U) := \Sigma(s,U) - \Sigma(s,T)$ and $A(s,T,U) := A(s,U) - A(s,T)$.

- The martingale measure is *unique* (cf. Eberlein, Jacod and Raible 2005).

Payoff of composition

Discrete tenor structure: $0 = T_0 < T_1 < \cdots < T_N < T_{N+1} = T^*$, $\quad \delta = T_i - T_{i-1}$.



Composition: investment of 1 currency unit at the LIBOR rate for N periods:

$$\prod_{i=1}^{N} \big(1 + \delta_i L(s_i, T_i)\big)$$

and a cap on the composition pays off:

$$\left(\prod_{i=1}^{N} \big(1 + \delta_i L(s_i, T_i)\big) - K \right)^+ .$$

Valuation of compositions

Using that $1 + \delta_i L(s_i, T_i) = \frac{B(s_i,T_i)}{B(s_i,T_{i+1})}$ and the forward measure

$$\begin{aligned}\mathbb{C}(T^*; K) &= \mathbb{E}_{\mathbb{P}} \left[\frac{1}{B^M_{T^*}} \left(\prod_{i=1}^{N} (1 + \delta_i L(s_i, T_i)) - K \right)^+ \right] \\ &= B(0, T^*) \mathbb{E}_{\mathbb{P}_{T^*}} \left[\left(\prod_{i=1}^{N} \frac{B(s_i, T_i)}{B(s_i, T_{i+1})} - K \right)^+ \right] \\ &= B(0, T^*) \mathbb{E}_{\mathbb{P}_{T^*}} \left[(\exp H - K)^+ \right],\end{aligned}$$

where the random variable H is defined as

$$H := \log \frac{B(0, T_1)}{B(0, T^*)} + \sum_{i=1}^{N} \int_0^{s_i} A(s, T_i, T_{i+1}) \mathrm{d}s - \sum_{i=1}^{N} \int_0^{s_i} \Sigma(s, T_i, T_{i+1}) \mathrm{d}L_s.$$

The price of a cap on the composition is calculated using Theorem 1 and Example 3, i.e.

$$\mathbb{C}(T^*;K) = \frac{B(0,T^*)}{2\pi} \int_{\mathbb{R}} M_H^{T^*}(R - iu) \frac{K^{1+iu-R}}{(iu-R)(1+iu-R)} \mathrm{d}u, \tag{30}$$

where the moment generating function H is

$$M_H^{T^*} = \mathcal{Z}^{\star} \exp \int_0^{T^*} \theta_s \Big(\Sigma(s,T^*) - z \sum_{i=1}^{N} \Sigma(s,T_i,T_{i+1}) 1_{[0,s_i]}(s) \Big) \mathrm{d}s. \tag{31}$$

- Analogous results in the *forward price* model driven by time-inhomogeneous Lévy processes – a LIBOR-type model based on the forward process

$$1 + \delta_i L(t,T_i) = F(t,T_i,T_{i+1}).$$

Concluding remarks

Wiener–Hopf factors: $\kappa(\alpha, \beta)$ is known explicitly only in special cases – Brownian motion, spectrally negative Lévy processes, phase-type processes ...

Further applications in credit risk: structural models ...

The main contents of this talk can be found in:

E. Eberlein, A. Papapantoleon (2007). Valuation of exotic and credit derivatives in Lévy models. Working paper.

W. Kluge, A. Papapantoleon (2007). On the valuation of compositions in Lévy term structure models. FDM Preprint 96.

A. Papapantoleon (2006). *Applications of semimartingales and Lévy processes in finance: duality and valuation*. Ph.D. thesis, University of Freiburg.

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