Approximation of good deal bound solutions

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ABSTRACT

The paper shows how to find approximate "good deal" bounds for European claims in incomplete markets. We consider markets where the incompleteness is caused by presence of jumps or a non-traded factor. The "good deal" bound solutions were first introduced by [1], who suggested to rule out not only prices which create arbitrage opportunities but also price processes with "too high" Sharpe ratios. Imposing a uniform bound B on Sharpe ratios of all the derivatives and portfolios in the market, they find highest and lowest prices subject to the imposed constraints. The theory was extended by [2] on the models where the incompleteness is caused by jumps in the underlying asset's price process. The bounds are shown to be the solutions of the appropriate stochastic optimal control problems. In a general case, good deal bounds cannot be computed explicitly, which enables us to use numerical finite-difference methods. The procedure would require even more computational time if the underlying is driven by a general marked point process, especially if we would like to compute solutions for several values of the bound B. Thus, to simplify the numerical procedure, we find a linear approximation of the good deal bound price, writing Taylor expansion of the good deal bound prices around the price given by the minimal martingale measure (MMM). We expand the good deal prices in the new variable y, which is defined as a square root function of the good deal bound B and some parameters of the model. The MM measure provides us with a canonical benchmark for pricing any derivative, it has simpler structure than good deal bound prices and is much easier to compute. In order to compute the approximated bounds we find PDEs to which the MMM price and the sensitivity of the option prices with respect to the new parameter y (evaluated at the MMM solution) satisfy. We show that the linear approximation works extremely well for the small deviations of the bound value from bound value which corresponds to the MMM solution.

References

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