Approximation of good deal bound solutions

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ABSTRACT

The paper shows how to find approximate “good deal” bounds for European
claims in incomplete markets. We consider markets where the incompleteness
is caused by presence of jumps or a non-traded factor. The “good deal” bound
solutions were first introduced by [1], who suggested to rule out not only prices
which create arbitrage opportunities but also price processes with “too high”
Sharpe ratios. Imposing a uniform bound $B$ on Sharpe ratios of all the deriva-
tives and portfolios in the market, they find highest and lowest prices subject to
the imposed constraints. The theory was extended by [2] on the models where
the incompleteness is caused by jumps in the underlying asset’s price process.
The bounds are shown to be the solutions of the appropriate stochastic opti-
mal control problems. In a general case, good deal bounds cannot be computed
explicitly, which enables us to use numerical finite-difference methods. The pro-
cEDURE would require even more computational time if the underlying is driven
by a general marked point process, especially if we would like to compute solu-
tions for several values of the bound $B$. Thus, to simplify the numerical
procedure, we find a linear approximation of the good deal bound price, writ-
ing Taylor expansion of the good deal bound prices around the price given by
the minimal martingale measure (MMM). We expand the good deal prices in
the new variable $y$, which is defined as a square root function of the good deal
bound $B$ and some parameters of the model. The MM measure provides us
with a canonical benchmark for pricing any derivative, it has simpler structure
than good deal bound prices and is much easier to compute. In order to com-
pute the approximated bounds we find PDEs to which the MMM price and the
sensitivity of the option prices with respect to the new parameter $y$ (evaluated
at the MMM solution) satisfy. We show that the linear approximation works
extremely well for the small deviations of the bound value from bound value
which corresponds to the MMM solution.
References
