Issuance Costs and Stock Return Volatility

by

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Introduction

▷ Cash-flows: $\mu dt + \sigma dW_t$

▷ no debt

▷ The firm is in activity as long as the cash reserve is positive.

▷ Firm can collect new funds issuing new shares at some cost

▷ Firm can distribute dividends at no cost

▷ Question: Implications of these modeling assumptions on Asset Pricing.

▷ Main result: Issuance costs imply heteroscedasticity in the stock prices. " When things go badly for the firm, its stock price will fall, and the volatility of the stock will go up."

The Model (1)

▷ Cumulative cash-flows

$$R_0 = 0 \quad dR_t = \mu dt + \sigma dW_t.$$

▷ Frictions

- Fixed and proportional issuance costs
- managerial inefficiencies

▷ Issuance policy

- dates at which new equity is issued: $(\tau_n)_{n\geq 1}$
- issuance proceed: $(i_n)_{n\geq 1}$
- Total issuance proceed: $I_t = \sum_{n\geq 1} i_n \mathbb{1}_{\tau_n \leq t}$
- Total fixed issuance costs: $F_t = \sum_{n\geq 1} f \mathbb{1}_{\tau_n \leq t}$

▷ Dynamics of the cash reserves up to $\tau_B = \{t \geq 0 \,|\, M_t < 0\}$

$$M_0^- = m, \quad dM_t = (r-\lambda)M_t dt + dR_t + \frac{1}{p} dI_t - dF_t - dL_t$$

The Model (2)

▷ Value of the firm for a given policy

$$v(m; (\tau_n)_{n\geq 1}, (i_n)_{n\geq 1}, L) = \mathbb{E}^m\Big[\int_0^{\tau_B} e^{-rt}\,(dL_t - dI_t)\Big],$$

▷Value function

$$V^*(m) = \sup_{(\tau_n)_{n\geq 1}, (i_n)_{n\geq 1}, L} \Big\{ v(m; (\tau_n)_{n\geq 1}, (i_n)_{n\geq 1}, L) \Big\}$$

▷ Questions

- value function,
- optimal issuance and dividend policies,
- resulting stock price process,
- testable asset pricing implications.

Benchmark: $\mathbf{p = 1,\ f = 0,\ \lambda > 0}$

▷ Firm value

$$V(m) = m + \mathbb{E}^m\left[\int_0^\infty e^{-rt}\left(\mu dt + \sigma dW_t\right)\right] = m + \frac{\mu}{r}.$$

▷ The pair (L, I)

$$L_t = m1_{\{t=0\}} + lt$$
$$I_t = (l - \mu)t - \sigma W_t$$

$$V(m) = \mathbb{E}^m\left[\int_0^\infty e^{-rt}\left(dL_t - dI_t\right)\right] = m + \frac{\mu}{r}$$

▷ Dynamics of stock price.

$S = \{S_t; t \geq 0\}$ ex-dividend price of a share in the firm
$N = \{N_t; t \geq 0\}$ number of outstanding shares of the firm

$$V(M_t) = N_t S_t$$

$$dI_t = d(N_t S_t) - N_t dS_t = -N_t dS_t = -\frac{\mu}{r}\frac{dS_t}{S_t}$$

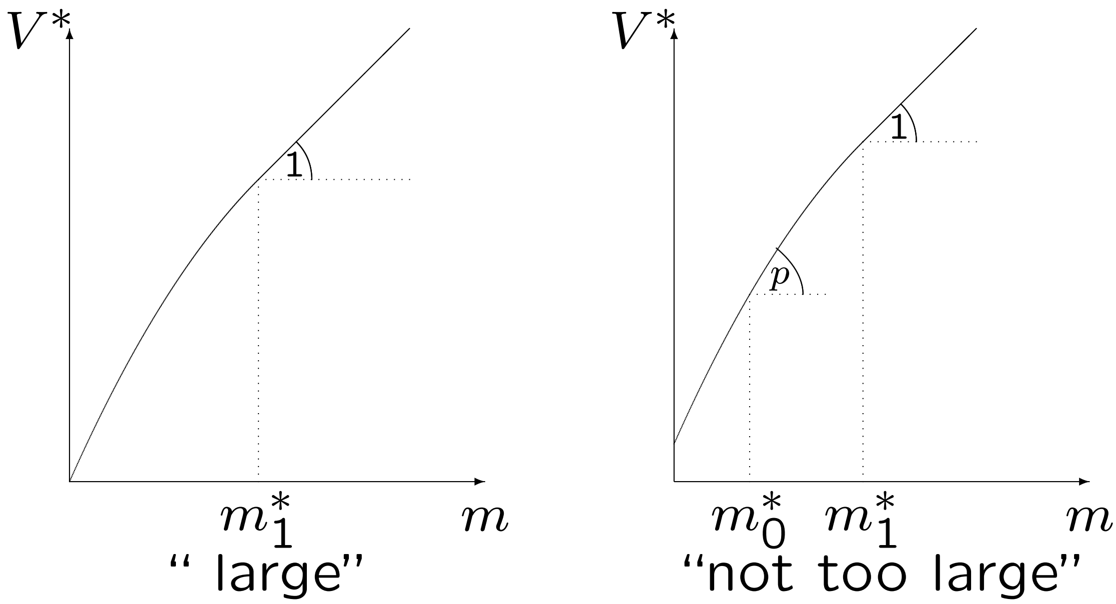
$$\frac{dS_t + dD_t}{S_t} = rdt + \frac{\sigma r}{\mu} dW_t$$

where D_t is the cumulative dividend per share process:

$$dD_t = l\frac{r}{\mu}S_t dt = \frac{l}{N_t}dt.$$

Value function

$$V^*(m) = \sup_{I_t, L_t} \mathbb{E}^m \Big[\int_0^{\tau_B} e^{-rt} (dL_t - dI_t) \Big]$$



▷ Cash reserve process M at the optimum.

- If issuance costs are "large": diffusion process that is reflected back each time it hits m_1^*, and that is absorbed at 0.
- If issuance costs are "not too large": diffusion process that is reflected back each time it hits m_1^*, and jumps to m_0^* each time it hits 0.

▷ Optimal issuance policy

- Firm value jumps from $V^*(0)$ to $V^*(m_0^*)$
- Each time M hits zero, the amount $V^*(m_0^*) - V^*(0)$ of new equity is issued.

Stock price dynamics (1)

$S = \{S_t; t \geq 0\}$ ex-dividend price of a share in the firm
$N = \{N_t; t \geq 0\}$ number of shares issued by the firm (non decreasing process)

- Stock price does not jump at optimal issuance dates: $S_{\tau_n} = S_{\tau_n -}$
- $V^*(M_t) = N_t S_t$

- $dI_t = d(N_t S_t) - N_t dS_t = S_t dN_t$
- $dS_t = d[V^*(M_t)]/N_{\tau_n} \ \ \forall t \in [\tau_n, \tau_{n+1})$.

Stock price dynamics (2)

Proposition. Between two consecutive issuance dates τ_n and τ_{n+1}, the stock price process S evolves according to:

$$\frac{dS_t + dD_t}{S_t} = rdt + \sigma(N_{\tau_n} S_t) dW_t,$$

where

$$\sigma(v) \equiv \sigma \frac{V^{*\prime}\left[(V^*)^{-1}(v)\right]}{v}$$

and D_t denotes the cumulative dividend per share process:

$$dD_t = \frac{V^{*\prime}(M_t)}{N_{\tau_n}} dL_t^{m_1 *}.$$

Consequences:

- Changes in the volatility of stock returns are negatively correlated with stock price movements.
- Changes in the volatility of stock prices are negatively correlated with stock price movements.
- Stock price cannot take arbitrarily large values.
- A reduction in issuance costs should lead to a fall in the volatility of stock returns.

Conclusion

We have:

- put at work a model in the vein of Jeanblanc and Shiryaev (1995) and Løkka and Zervos (2005)
- linked corporate finance and asset pricing

Main results:

- Volatility of stock returns
- Volatility of stock prices
- Transaction costs $\rightleftharpoons$ volatility of financial markets.

Stock price dynamics

Proposition. The process N modelling the number of outstanding shares is punctual and defined by:

$$N_t = \begin{cases} 1 & 0 \le t < \tau_1, \\ \left[\frac{V^*(m_0^*)}{V^*(0)}\right]^n & \tau_n \le t < \tau_{n+1}. \end{cases}$$

Proposition. Between two consecutive issuance dates τ_n and τ_{n+1}, the stock price process S evolves according to:

$$\frac{dS_t + dD_t}{S_t} = rdt + \sigma(N_{\tau_n} S_t) dW_t,$$

where

$$\sigma(v) \equiv \sigma \frac{V^{*\prime}\left[(V^*)^{-1}(v)\right]}{v}$$

and D_t denotes the cumulative dividend per share process

$$dD_t = \frac{V^{*\prime}(M_t)}{N_{\tau_n}} dL_t^{m_1 *}.$$

Comparative statics

Proposition

- The elasticity of the value of the firm with respect to its cash reserves,

$$\epsilon^*(m) = \frac{mV^{*\prime}(m)}{V^*(m)}; \quad m \geq 0,$$

  is an increasing function of the issuance costs p and f.

- The volatility of stock returns as a function of the firm's valuation,

$$\sigma^*(v) = \sigma \frac{V^{*\prime}((V^*)^{-1}(v))}{v}; \quad V^*(0) \leq v \leq V^*(m_1^*),$$

  is an increasing function of the issuance costs p and f.

$\Longrightarrow$

- A reduction in issuance costs should reduce the responsiveness of firm's valuations to changes in their cash reserves.
- A reduction in issuance costs should lead to a fall in the volatility of stock returns.

Value function

Road map:

- Write a system of variational inequalities that the value function V^* should satisfy.
- Show that this system has a unique regular solution.
- Establish that this solution is indeed the optimal value function.

Value function (1)

▷ Heuristics

$$V^*(m) \geq V^*(m-l)+l$$
$$V^{*\prime}(m) \geq 1$$

$$V^*(m) \geq V^*(m+\frac{i}{p}-f)-i$$

$$V^*(m) \geq$$
$$\mathbb{E}^m\Big[e^{-r(t\wedge\tau_B)}\,V^*\Big(m+\int_0^{t\wedge\tau_B}[(\mu+(r-\lambda)M_s)ds+\sigma dW_s]\Big)\Big]$$
$$-rV^*(m)+\mathcal{L}V^*(m) \leq 0$$

Value function (2)

▷ Guess

- Issuance policy

$$V^*(0) = \left[\max_{i\in[0,\infty)}\left\{V^*\left(\frac{i}{p}-f\right)-i\right\}\right]^+,$$

$$V^*(0) = \left[\max_{m\in[-f,\infty)}\{V(m)-p(m+f)\}\right]^+$$

- Dividend policy $m \geq m_1^*$,

$$V^{*\prime}(m) = 1.$$

  V^* is postulated to be twice continuously differentiable over $(0,\infty)$,

$$V^{*\prime\prime}(m_1^*) = 0.$$

Value function (3)

▷ Variational system: Find (V, m_1)

$$V(m) = 0;\ m < 0, \tag{1}$$

$$V(0) = \left[\max_{m \in [-f,\infty)} \{V(m) - p(m+f)\}\right]^+; \tag{2}$$

$$-rV(m) + \mathcal{L}V(m) = 0;\ 0 < m < m_1, \tag{3}$$

$$V(m) = \frac{\mu + (r-\lambda)m_1}{r} + m - m_1; \quad m \geq m_1. \tag{4}$$

▷ Solving the system

Fix $m_1 > 0$, V_{m_1} solution to:

$$-rV_{m_1}(m) + \mathcal{L}V_{m_1}(m) = 0; \qquad 0 \leq m \leq m_1,$$
$$V'_{m_1}(m_1) = 1,$$
$$V''_{m_1}(m_1) = 0.$$

V_{m_1} solution to (1)-(4) linearly extended to $[m_1, \infty)$.

Value function (4)

Lemma
(i) $\exists\,!\ \hat{m}_1 \;\; V_{\hat{m}_1}(0) = 0$,
(ii) $\exists!\ \tilde{m}_1 \;\; V'_{\tilde{m}_1}(0) = p$,
(iii) $m_1 > \tilde{m}_1$, $\quad \exists\,!\ \ m_p(m_1) \ \ s.t \ \ V'_{m_1}(m_p(m_1)) = p$.

Proposition

(i) If $\max_{m\in[-f,\infty)}\{V_{\hat{m}_1}(m) - p(m+f)\} = 0$ then,
$(V, m_1) = (\hat{V}, \hat{m}_1)$ where

$$\hat{V}(m) = \begin{cases} 0 & m < 0, \\ \\ V_{\hat{m}_1}(m) & m \geq 0. \end{cases}$$

(ii) If $\max_{m\in[-f,\infty)}\{V_{\hat{m}_1}(m) - p(m+f)\} > 0$ then,
$(V, m_1) = (\bar{V}, \bar{m}_1)$ where

$$\bar{V}(m) = \begin{cases} 0 & m < 0, \\ \\ V_{\bar{m}_1}(m) & m \geq 0. \end{cases}$$

with $\bar{m}_1 \in (\tilde{m}_1, \hat{m}_1)$ such that
$V_{\bar{m}_1}(0) = V_{\bar{m}_1}(m_p(\bar{m}_1)) - p[m_p(\bar{m}_1) + f]$.

Value function (5)

▷ For any admissible issuance and dividend policy $((\tau_n)_{n\geq 1}, (i_n)_{n\geq 1}, L)$,

$$V(m) \geq v(m; (\tau_n)_{n\geq 1}, (i_n)_{n\geq 1}, L); \quad m \geq 0.$$

▷ Unique solution $(M^*, L^*) = \{(M_t^*, L_t^*); t \geq 0\}$ to

$$\begin{aligned} M_t^* &= m + \int_0^t (\mu + (r-\lambda)M_s^*)\, ds + \sigma W_t \\ &\quad + \sum_{n\geq 1} m_0^* 1_{\{T_n^* \leq t\}} - L_t^*, \\ M_t^* &\leq m_1^*, \\ L_t^* &= \int_0^t 1_{\{M_s^* = m_1^*\}}\, dL_s^*, \end{aligned}$$

for all $t \in [0, \tau_B^*]$, where $\tau_B^* = \inf\{t \geq 0 \,|\, M_t^* < 0\}$ and the sequence of stopping times $(T_n^*)_{n\geq 1}$ is recursively defined by:

$$T_n^* = \inf\{t \geq T_{n-1}^* \,|\, M_{t-}^* = 0\}; \quad n \geq 1$$

where $T_0^* = 0$.