Indifference pricing of a life insurance portfolio with systematic mortality risk in a market with an asset driven by a Lévy process

Łukasz Delong

Warsaw School of Economics,
Institute of Econometrics, Division of Probabilistic Methods,
Niepodległości 162, Poland
lukasz.delong@sgh.waw.pl

ABSTRACT

We investigate the problem of pricing and hedging of life insurance/pension liabilities. We consider a payment process $(P(t), 0 \leq t \leq T)$ of the form

$$dP(t) = N(t-)cdt - bdN(t) + N(T)B(T)d\mathbf{1}\{t \geq T\}, \quad P(0) = n_0 B(0),$$

where $c, b, B(0), B(T)$ denote premiums and benefits, specified by a contract, and $(N(t), 0 \leq t \leq T)$ counts the number of survivors in a portfolio, with $N(0) = n_0$. An insurer can invest in a financial market consisting of a risk-free asset, with constant rate of return, and a risky asset, which dynamics is modelled by a Lévy process. We take into account systematic mortality risk and model mortality intensity $(\lambda(t), 0 \leq t \leq T)$ as a diffusion process with the dynamics

$$d\lambda(t) = \eta(t, \lambda(t))dt + \theta(t, \lambda(t))d\bar{W}(t).$$

In this incomplete market, the principle of equivalent utility is chosen as the valuation rule. In order to solve our optimization problems we apply techniques from the stochastic control theory. An exponential utility is considered in details. We arrive at three pricing equations and investigate some properties of the obtained premiums. In particular, an estimate of the finite-time ruin probability is derived. From the point of mathematical finance, our results can be applied to solve optimal investment problems with uncertain time horizon, where the intensity of exiting the market is a stochastic process.